Complex Endogenous Growth Model and Its Applications

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Abstract

In this paper, social complex systems are applied to the endogenous economic growth model. It is discussed that the economic growth put forward in Romer's endogenous growth model is not continuous, but is rather in the functional shape of the bifurcations known to the natural sciences. The author has proposed that the level of development (factor A in a classic growth model) should be the superposition of the structures of science and real economy in the observed country. The conclusion is drawn that closer correspondence between business and R&D results in (i) higher economic output for the country; and (ii) a greater resilience to external shocks. This is especially important for small open economies with distinct cultures and languages (e.g. Estonia), that should simultaneously increase its citizens well-being, but must account for exogenous economic and technological factors. It is concluded that under the assumptions of complexity, a state's economic growth model should (i) strive for horizontally and vertically diversified risks, as well as (ii) assume that none of its society's members is left behind in terms of education, employment, and general wellbeing.

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1. Properties of Social Complex Systems

1.1. Introduction to Social Complex Systems

In his contribution from 2010, professor Jüri Engelbrecht (Engelbrecht, 2010) gave an elegant overview of complex systems. He discussed that there is no precise definition of complex systems; however, there are generally accepted properties that complex systems tend to possess. Although his focus was on natural systems, he did not rule out social systems. In his previous papers, the present author has discussed the social complex systems (Kitt, 2011) and has also described some of their applications (Kitt, 2014a, 2014b). Numerous applications of social complex systems are also given in the works of Miller and Page (2007) and Mitchell (2011). An overview of complexity economics is provided by Elsner (2015), Durlauf (2012) and Foxon (2013). What follows is a short overview of social complex systems and their properties, which are given in order to motivate a complex extension of the economic growth model.

First and foremost, complex systems are composed of multiple constituents that interact with one another. Furthermore, such kinds of systems are prevailingly hierarchical. The global economic complex system comprises eight billion reciprocally influential components or economic agents. While the component elements of many physical and mechanical systems are more or less equal in size, the size of just a few companies (e.g. Coca-Cola, Microsoft), or individuals (e.g. Bill Gates), is many orders of magnitude larger than the majority of other companies or individuals.

An intrinsic feature of complex systems lies in the fact that these processes are not obtainable by summing, multiplying or dividing the qualities of their initial particles. In other words, the principle of linear superposition, which could at any moment enable a reduction of the features characterising a particular system to combinations of its initial particles, is not applicable to complex systems. Similarly, deriving the reaction of the system on external influences or internal processes merely from knowledge about the behaviour models of the initial particles, is also not possible. In short, the behaviour of complex systems is, by and large, definable by nonlinear relations. This quality in the financial world is, for example, reflected by the observation that on the vast majority of trading days the volatility of stock prices is low; however, the overall performance of the stock market is typically achieved in a few trading days, when the markets demonstrate very high price movements. This phenomenon is studied using a sub-branch of statistical mechanics, econophysics, and the results can be obtained from Mantegna and Stanley (2000), Kitt (2005) and Stanley et al. (2008). Classical statistics proclaims them extreme incidents that should by no means be taken into consideration for forecasting future events and, in fact, it's better if they are completely ignored. If, however, those extremities were actually cut out of the picture, the whole meaning of a stock exchange system would get lost. Thus, when dealing with complex systems, we have to reckon with all kinds of contingencies, leaps, extremities and breaking points, whose existence would be out of the question in a simple world that follows the rules of linear superposition.

Complex systems may be self-organising. On these occasions, elementary particles generate principally new structures, non-existent at some earlier moment, whose formation has not been obviously predictable. New components may also occur via certain types of particle interactions: combining, convergence as well as division. The characteristics of those particles differ from the qualities of initial particles. In the context of social complex systems, the formation of limited liability companies or public institutions can be ascribed as the same process. Indeed, the behaviour of company and/or institution may differ from the behaviour of any of the employees or founders of that economic unit.

Complex systems have a memory. In mechanical systems, that type of memory is involved in processing quantities that obey simple rules like impulse, temperature or pressure, whose modifications can be calculated independently for every component of a system. When addressing economic processes, it would be deeply misleading to surmise that they are mutually independent, or acting merely by virtue of inertia. What happens tomorrow is in an extremely convoluted process, dependent on the occurrences of yesterday. Complex systems are not just temporally changing they are also adaptive. Therefore, when something has happened they commit it to memory and adjust themselves to different circumstances. Consequently, changes in the economy are thermodynamically unstable. In the same way as in complicated aero-hydrodynamic systems like the Earth's atmosphere or oceans, local modifications in social complex systems have the potential to turn global. This quality is generally known as the so-called butterfly effect.

Complex systems function on the borderline between order and chaos. Often enough, a considerable number of them follow simple rules for a lengthy period of time. Inclusion of such a mathematical category as a chaotic attractor into the category of structured patterns is justified. In some systems, or their components, the motion of particles is entirely random. It is important to understand that completely desultory systems are non-existent, but they might be lacking any form of distinctive order.

Given the globalisation and interdependencies of global economic systems, the complexity of social systems is growing at an immense speed – the systems are progressively complex.

1.2. The Rise of Power Laws in Economic Systems

Numerous economic complex systems have been empirically explored. Econophysics-based analyses of stock markets, using methods from nonlinear dynamics, have clearly indicated that power laws, as well as memory, etc. apply to processes in financial markets. For the author, there is no apparent reason why, for example, standard deviation of price fluctuations, should be the characteristics of risk. A curious aspect may be found in the detail that whenever talking about power laws, physicists refer to the names of social scientists. For example, George Kingsley Zipf (1902-1950), whom physicists refer to when discussing the power law (Zipf, 1949), was a professor of linguistics at Harvard University. He described a phenomenon in the English language, where the language corpus is distributed in such a way that a large number of words are rarely used and a few words are heavily used. Since then, the frequency distribution of words has been labelled as 'Zipf's law'. Another name frequently mentioned in connection with this topic is Vilfredo Pareto (1848-1923), who noted that in Italy, Peru and several other countries an extremely small group of people own a huge amount of land (Pareto, 1896-1897).

It appears that such simple property is very widespread: references in academia, war casualties, and income distributions are just a few examples that follow the so-called power law. A good overview of this topic is given by Gabaix (2016). Mathematically, the power-law is written as follows: $P(x)\ \alpha\ x^{-\alpha}$, where $P(x)$ denotes the cumulative probability distribution function of variable x. The power law can also be interpreted simply by noting that the number of system members obeying property x decreases in power of α. The power law leads to scale invariance; i.e. to the system that has no characteristic scale to describe it. For example, the power law in stock price movements leads to infinite standard deviation (or volatility) that is frequently used as a proxy to risk. Therefore, the use of linear characteristics (such as standard deviation) can be not only misleading, but also dangerous.

While quite a number of super-universal laws have been established regarding the nature – natural laws that are functioning regardless of time and space, e.g. the laws of energy and momentum conservation, or the law of increasing entropy – no similar universal laws are available for social systems. There is no reason why a generic model that was working yesterday should remain functional tomorrow. Therefore, whenever applying models in economic forecasting or planning, one must consider that there are no super-universal laws of society that work unconditionally in all societies regardless of time and place.

However, the lack of super-universality does not hinder the use of mathematical models under certain pre-defined assumptions. If the assumptions become invalid, the model ceases to work. The increased spreading of power-laws in economics and society can be ascribed to the Barabási-Albert model (Barabási and Albert, 1999 and references in Albert and Barabási, 2002). This model states that a network becomes scale-free – i.e. degree distribution of network nodes obeys power law – if, and only if, following two qualities are present in its dynamics: (i) growth (that can also be with a minus sign, i.e. a decline) and (ii) preferential attachment.

1.3. Open Society as the Trigger of Complexity

An immense number of social (complex) systems comply with the aforementioned prerequisites. The vast majority of economic systems indicate preferential attachment. They need not be conscious, but people prevailingly prefer a specific brand or a particular book, or they pick out their favourite friends on Facebook, etc. The Barabási-Albert model hints why many social systems are complex systems. Although we have no philosophical ground yet to assert that this is a super universal model, it offers at least a certain rational explanation. In reality it is impossible to bring forth any examples of a social system that is not complex – there are no social systems that would not be undergoing modification or having preferential attachments.

It can be shown that Western liberal democracies trigger complexity and power laws in their societies and economies. This claim can be motivated as follows: first and foremost, the citizens of Western liberal democracies obey political and economic free will. They have democratic institutions whose policies can be changed without blood, based on the public mandate (i.e. elections). The citizens of the West also have economic free will – the right to consume and spend in whichever way they please. Of course, the citizens should also bear political and economic consequences, as the free will without responsibility is simply anarchy. In turn, the economic freedom generates preferential attachment as well as growth (or contraction) and therefore the rise of complexity is triggered. Karl Popper (1902-1994) has elaborated on this through the concept of Open Society (Popper, 1945). Popper also discusses the ethical aspects of collective goals in society, given that history has no meaning (N.B: the power laws lead to the infinite variance of events). However, where Popper and other liberal thinkers, such as Friedrich Hayek, Ludwig von Mises and Milton Friedman generally do not appreciate any redistribution of wealth, John Rawls (1921-2002), in his Theory of Justice (Rawls, 2001), allows for the redistribution of wealth in society only on the condition that it benefits the societies poorest. Rawls postulates that if a society were genuinely honest, it would have to offer a reality where any two members of the society (king and pauper) readily consent to exchange their roles. Rawls admits a slightly hypothetical nature of that statement. The king should be motivated to raise the pauper's standard of living.

Due to the complex nature of social systems, one might easily reach the conclusion that social planning and economic forecasting are not only misleading, but also frequently dangerous. Popper maintains that history has no objectives and the future is unknown for everyone alike. Another aspect to be considered is the reality whereby Excel tables may be trusted to accurately show those things that have been entered, but not necessarily the right things. Centralised planning is basically not possible, because there is no reason why it should work – who is that super-universal person, capable of deciding, which sectors should be chosen, etc. There is no such super-universal law that would motivate centralised thinking.

2. An Endogenous Economic Growth Model based on the Theory of Complex Systems

Generally, an arbitrary economic growth model is based on the assumption that to the first approximation economic output (at time *t*) *Y* equates to capital input *K*, and labour input *L*. If proportionality constant *A* denotes effectiveness of resource utilisation, the growth theory equation is presented in general bilinear form, which says that the output at time t is expressed as the product of the aforementioned three factors: $Y=AKL$.

2.1. Endogenous Growth Models

The models of economic growth can be formulated in many different ways (Romer D., 2012), but apparently many influential models of endogenous growth (Arrow, 1962; Lucas, 1989 and Romer P., 1986) can be written as follows:

$$Y(t) = A(t)F(K,L) \tag{1}$$

Capital input *K* and labour input *L* are slow to change; therefore, it is possible to achieve any faster growth rates in output only by improving the effectiveness of resource utilisation (i.e. constant *A*). There are many discussions in economic theory, as well as policy suggestions on innovation and growth – Howitt, 2007, for example. As with many economic theories, the mathematical representation of growth theories may yield very complicated equations, but in essence they model the continuous dynamics of the underlying parameters (e.g. household budget, productivity, etc.).

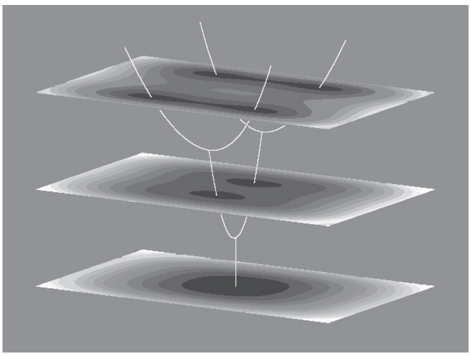
However, the idea behind output levels is fairly simple and it is rooted to the effectiveness of resource utilisation in a particular branch of the economy. Nowadays that is roughly equal to how proficiently that branch has succeeded in putting R&D outcomes into practice. If everything is reliant on technologies deriving from the 19th century, that industry is surely going to fall apart in the near future. On the other hand, it will be among the best if it has reasonably employed the latest achievements in science and technology. As different economic sectors may, at certain times, be in drastically different phases regarding the implementation process, the intensity of the economy as a whole is dependent upon the input made by individual branches (or business structure), as well as the extent of incorporated R&D in those industries. Hence, in an unexposed mode this constant contains information about the dimensions of business activity, as well as the structure of R&D and correspondence between them.

Aghion et al. (Aghion, 2013 and references therein) put forward an extensive study of the Schumpeterian growth model. In their papers, they motivate and empirically verify the importance of innovation, as well as democracy and openness, in achieving higher levels of growth.

2.2. Non-Linear Bifurcated Growth Patterns

The analysis of the non-linear effects of financial time series can be rooted back to 1960s and associated with Benoit Mandelbrot (Kitt, 2005 and references therein). According to the author's best knowledge, in a general economic context, the pioneering approach seems to come from professor Tõnu Puu in 1989 (Puu, 1989; Puu, 2013 and references therein). In his seminal book, "Arts, Sciences and Economics – A historical safari" (Puu, 2015), Professor Tõnu Puu discussed bifurcations that are likely to govern the evolution of culture and technology. More specifically, by defining culture as art plus science, he discusses the evolution of social and material products. Kelvin Lancaster has studied consumer goods (Lancaster, 1971) and their evolution. Any consumer good (or any good in general) has certain features that are important in this context. If one lists all possible features of items ever created, each particular item will have some "high scores" in the categories for which the item has been created. For example: a knife scores high in cutting, but low in sewing; whereas a ball scores high in group exercise, but low in nutrition. Such somewhat artificial construction allows Lancaster to create a multidimensional space, where all of the items can be placed. Puu elaborated on the Lancaster space and postulated that there is continuous progress in terms of complexity of the items, but the progress is not linear, but bifurcated. In economic terms, if there is a demand for certain new parameter (or a new feature), then some of the existing parameters are most likely bifurcated and qualitatively new features are created. In figure 1 the illustration of this bifurcate approach is presented.

Figure 1. Stylised model by Tõnu Puu: How Sequential Bifurcations May Create an Evolutionary Tree in Lancaster Variable Space.



In the context of endogenous growth model, the bifurcations play very important role. Namely, if one assesses the functional shape of parameter $A(t)$ in equation (1) it should be discontinuous and contain "jumps". This is obvious as acquired competences (both knowledge and technological) evolve according to the bifurcation theory. In practise, acquired new machinery (in business) or knowledge (in culture) will qualitatively shift the output of production. This is also supported by empirical evidence where technological progress has been estimated in different countries over longer time periods. As innovation is not predictable by definition, it is not known in which direction the bifurcation tree evolves. Therefore, both cultural and business environments are prone to competition and continuous change.

2.3. Endogenous Growth Arising from Science and Business

The author stands firm in his belief that each and every economy will be sustainable in the long-term if, and only if, it drives growth from internal sources. This does not imply that in the short-term there should be no external subsidies and/or borrowings. On the contrary, given the interconnectedness of global capital flows and R&D, Schumpeterian "creative destruction" should be always treated as an exogenous factor. However, this is exactly why risk diversification is called for in all countries. One cannot assume any of the long-term competitive advantages without continuously investing into internal resources. By "internal resources" the author implies that long-term growth is primarily the result of investments into human capital, scientific and technical competences. As this is also the definition of endogenous growth theory (Romer, 2012), in what follows the author looks for the policy suggestions for sustainable long term growth.

Similarly, to Lancaster variable space, the activity space is constructed for an arbitrary country (or region). The activity space for economic activities contains a finite number of areas where this territory is active (i.e. creates output). The activity space for scientific activities can be similarly defined for scientifically active areas. If there is a match between activities (i.e. both, science and economy are present), the variable spaces also match. For simplicity reasons, it is assumed, that the country has N different generic activity areas that could be either scientific or economic or both, creating the N-dimensional activity space.

Further, a vector $E= [w_{e1}, w_{e2}, \ldots, w_{en}]$ is defined to denote the structure of economic (or business) activity, where wei denotes the fraction of i-th economic activity from total economic output. By definition $\sum w_{ei}=1$.

Similarly, a vector $S= [w_{s1}, w_{s2,} \ldots, w_{sn}]$ is defined to denote the structure of scientific (or R&D) activity, wsj denotes the fraction of i-th scientific activity from total scientific output. By definition $\sum w_{si}=1$.

Thus, effectiveness of resource utilisation $A(t)$ is expressed in the form of a certain function pertaining to the scalar product of economic and R&D structure:

$$A(t) = f(E \times S^T), \tag{2}$$

where functional form of $f(x)$ corresponds to the evolutionary tree in Lancaster variable area, according to the model suggested by Tõnu Puu. Substituting (1) and (2) the total output can be written as:

$$Y(t) = f(E \times S^T)F(K,L). \tag{3}$$

The application of the above-mentioned model leads to a simple but significant conclusion: the closer the correspondence between economy and science (or the scalar product of vectors denoting them), the higher the attainable level in the evolutionary tree and the better the output of the economy. Geometrical interpretation of that concept is even simpler: from

Cauchy inequality, the aforementioned product has maximum value when the vectors are in the same direction, i.e. business and R&D are focussed on the same targets. The figure we get by multiplying the values reflecting economic and scientific structures, characterises something really tangible, that is to say, economic effectiveness – parameter A in the above equation. In other words, the economy is maximised under conditions where the intersection of economic sectors and R&D structures has the largest possible size.

Recently there have been a lot of publications and public discussions on so-called "smart specialisation" (Varblane et al., 2012; OECD, 2013; Foray et al., 2011 and references therein). It is also closely related to the Schumpeterian "creative destruction", elaborated on by Aghion et al. (2013). As the essence of smart specialisation is to leverage upon achieved successes in science and economy then the proposed model serves as a quantitative representation of smart specialisation. However, it must be emphasised that the dynamical and ever-evolving structure of Lancaster space does not allow for long stability of such smart specialisation. Global innovation (or bifurcation in Lancasterian space) does not follow any of the policy setting time frames. Therefore, the precondition for successful smart specialisation is very dynamic policy setting that allows the dynamic redirection of resources in a very short time frame. On the same grounds, the research funding policies should also account for rapid changes in science, but also in the surrounding economy.

3. Applications

In this section the endogenous growth model is applied in an Estonian context. Estonia has a small, open economy. It has a small and slowly declining population, with a unique cultural space based on the Estonian language. The policy discussions of Estonia (and many other similarly small states with their own unique language) should account for preservation of the culture and language. There are three applications or policy suggestions presented in this section. First, the role of universities in society is discussed, with the aim of devising policy suggestions that lead to increased economic growth. The second application focuses on the selection of growth areas: how to achieve diversification of risks with limited means. The third application focuses on a vertical diversification – job creation at various income levels. In the fourth subsection, the three applications are combined into a single, endogenous economic development model for Estonia.

3.1. The Economic Goal of Universities in Society is to Facilitate Sustainable Growth

In the literature, higher economic growth is *ceteris paribus* associated with higher innovation and better education. Therefore, it is useful to ask about the purpose and goals of universities and other similar institutions in achieving higher, country-level aggregated economic output. In general, universities pursue two goals: (i) they provide higher education and train specialists for the labour market and (ii) they pursue scientific research. Therefore, the role of universities is dual in nature – to act as an R&D institution as well as an educator of specialists for the national economy. Many of the policy discussions focus on achieving the optimal mix of those goals. What follows is a discussion of the policy choices devised in Section 2 from a growth model perspective.

First, it is assumed that a university focuses purely on scientific results. In this case, it may acquire a lot of patents and therefore facilitate the innovation in the country and presumably uplift the innovation parameter $A(t)$ in equation (1). However, the latter parameter $F(K,L)$ has very limited possibilities to leverage acquired innovation as there are no trained specialists to do so. If the universities do not train specialists for the general economy (both public and private sector), the vast majority of the labour force shall be less educated and therefore less qualified. Alas, students who do not qualify as top researchers have very limited opportunities outside of academia. The weak training of specialists will immediately hinder capital and labour productivities in a growth model (parameters K and L). Finally, a country with a weak economy cannot probably finance such universities, which leads to complete dependence on external sources of funding. This, by definition, should not be viewed as an inferior solution, but it clearly carries a high societal risk.

Secondly, the opposite is assumed: universities focus only on training specialists for the economy. This perhaps helps the economy, but hinders innovation and society lacks the critical power to reach new levels in the Lancasterian development space. It will always lag behind those who have a higher R&D that can aid financially, through the development of patents. Lack of R&D capabilities, or lack of innovation, will keep the society in low productivity mode, which will eventually become a drag to welfare or citizen living standards.

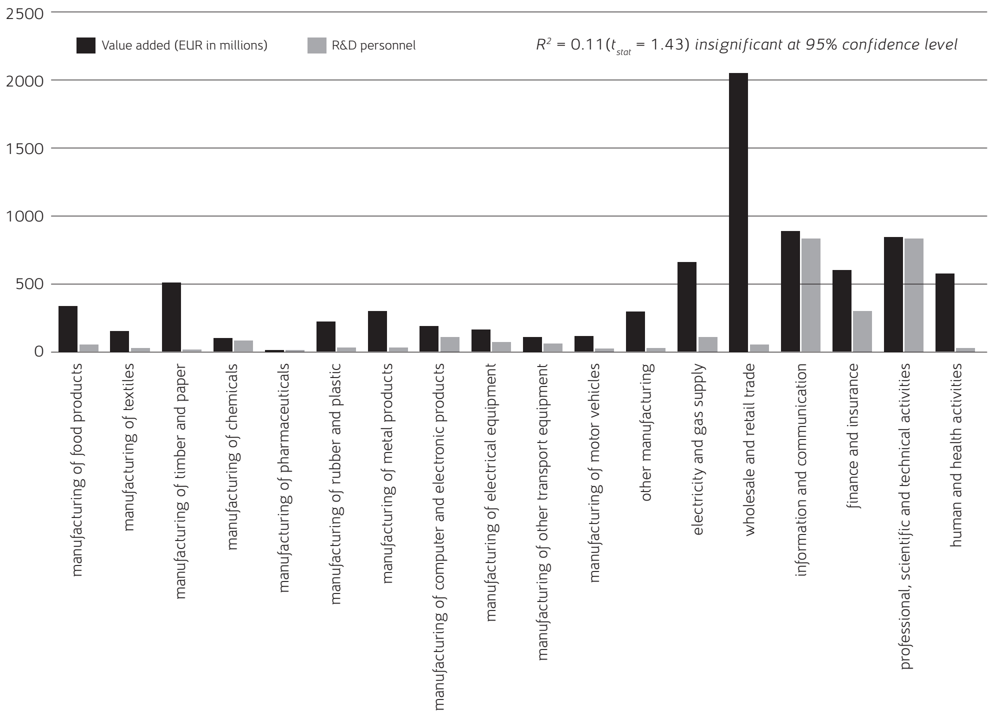
It is obvious that neither of those extremums seem to be viable ideas – both of them hinder the global optimum of national economic wellbeing. In a small cultural space, such as Estonia, this may lead to increased emigration and therefore is a direct threat to the existence of the culture and language. This can be generalised as follows: the economic goal of universities in society is to facilitate sustainable growth.

Therefore, the only reasonable policy suggestion from equation (3) is to maintain the balance between scientific ambitions and specialist training. Moreover, each university can, and should, leverage their existing strengths and only pursue scientific ambitions in the areas where specialist training is present and vice versa. Specialists should preferably be in the areas where the university (or country) already possesses novel research.

From a business point of view, it is not important how much a company invests into R&D. What is considerably more important is the rate of return on those R&D investments. This does not imply that all of the investments into scientific research should yield returns. However, there is positive correlation between the economic output of a country and its R&D investments (as predicted by equation (3)). Unfortunately, the discrepancy between Estonian R&D and business structures is large. According to Figure 2 below, there is a statistically insignificant correlation between economic value added and R&D employment by economic sectors. This claim has also been frequently proposed by Professor Urmas Varblane (Varblane, 2012; Karo et al., 2014).

From the discussion above, an obvious policy suggestion can be derived: – to interconnect frontline research and industry in a slightly more reasonable way –, the economy would become able to provide a considerably higher level of value added and reach higher growth rate of gross domestic product.

Figure 2. Added Value and the Amount of People Involved in R&D



Source: Author's calculations based on Statistics Estonia's data (2013)

3.2. Diversified Risks with Smart Specialisation

The previous subsection raises the following obvious question: assuming that it is possible to align the economy and science (as suggested in subsection 3.1), what are the most suitable areas to focus on? After all, since Ricardo's time basic economic theory suggests that everyone should focus on the areas where they have the best comparative advantage. However, from a complexity perspective, resilience is built by diversifying risks. So, the real question for small economies is: since it is not realistic to develop all economic areas, where should one focus? This subsection will show that specialisation through centralised planning is probably the worst idea for any country and that specialisation should only be pursued in areas where some sort of competitive advantage already exists.

To begin with the most recent studies, Luciano Pietronero and his colleagues have empirically shown that countries with the least specialisation produce a higher aggregate economic output (Pietronero et al., 2013). From the discussion of complex systems above, there is a strong argument that diversification of risks should be a pre-condition for each and every economic policy. This has also been discussed in previous papers written by the author (Kitt, 2014a; 2014b). The Ricardian question of specialisation can therefore be rephrased to: how can we keep the risks diversified, given the limited economic and scientific resources?

Canadian business strategist, Don Tapscott, coined an interesting term – the Internet economy. The Internet is based on distributed architecture operating as a network. Whenever a server falls short, the Internet as a system continues its work unhindered. The same idea should be applied to economy – if one company happens to fail, the economy as a whole should maintain its functionality. Concentration risk in the economy is the biggest risk a small country could take. Therefore, the best indicator for a viable business idea or a scientific area is a large number of entrepreneurs and scientists. If one fails, the rest are left untouched.

If diversification across sectors is desired, then equation (3) can also be interpreted in a way that the lack of either scientific or economic capacities ($w_i=0$) could be artificially compensated. In other words, the lack of certain industries or scientific capacities could be compensated by establishing such capacities using state funding. But, given the complex background of economic systems, the author is highly sceptical of such measures. There is probably a good reason why entrepreneurs and scientists have found no perspective for development in that particular area. Therefore, the policy suggestion is to facilitate favourable environment for positive evolutionary leaps in business, R&D and, preferably, in their collaboration and concurrence, but to not artificially create preference areas. The same can be concluded also from the point of smart specialisation: according to the equation (3), R&D investments are, as a rule, most effective in those areas, where some kind of comparative advantage has already been obtained, provided that they also have sufficient potential for increasing the capacity or even the proportion of their sectors. To summarise: (i) centrally planned concentration is a risk that the state cannot afford to take for longer periods; but (ii) in order to gain from an endogenous growth model, the knowledge transfer between science and business should be facilitated.

A small open economy cannot isolate itself from the external environment. On the contrary, small economies have much to gain from their openness, tolerance and variety of ideas. Given the complex environment of innovation and trade, the openness to international trade and R&D is crucial to keep pace with international trends. Further, the lack of resources could, and should, be solved via external recruitment or attraction: attraction of talents, physical capital and skilled labour are some to mention. This should give a strong signal to trade and foreign employment policy setters. However, in order to move up in the value chain (or Lancasterian evolution tree, or wellbeing of society) it is also important to build up the knowledge corpus domestically and with domestic resources.

There is an obvious flaw in the above argument: namely, what happens, if the country or region lacks any of the competitive advantages in business or science? Given the missing centralised planning unit under assumption of complexity, the organisation of global (or regional) economy (science) can easily yield such a result. In the author's opinion this should never be compensated by a closed and artificially created environment. Vice-versa, advantages should be created by opening up to external resources and step-by-step innovation, as described in Aghion et al. (2013).

It is well known that EU grants play an important role in the funding of Estonian science. It has been discussed about the future of Estonian science with potentially declining EU support. The author is in firm belief that all supports and grants should always be treated as temporary, with a limited lifetime. The effects of a decline in foreign aid largely depend on how efficiently this money has been previously invested. An example can be borrowed from the financial crises in 2008-2009. The Estonian economy was dependent on foreign loans. At some point these disappeared, along with the domestic demand, triggering a spike in non-performing loans. Such a trail of events sharply raises the question as to whether the loans or subsidies had been invested wisely during the years preceding the crises. Exactly the same question may be asked of the current moment in time – can we really say that at the point when the EU subsidies are terminated that the received subsidies have been reasonably invested? Have we established a scientific base, or a comprehensive conglomeration of knowledge that would enable us to maintain the effectiveness of the Estonian economy at a competitive level, even under conditions of limited financial resources (additional capital unavailable) and an exhausted labour force? The recovery from the past crisis has actually gone pretty well. It shows that majority of the investments made in the commercial sectors appeared to be self-sustainable and profitable.

To conclude, small open economies are the most resilient if they have a well-diversified economy, an Internet economy across the economic sectors and within specific sector. Given the interconnectedness of the global economy, small countries should be especially open to adopt new ideas (scientific or business) and markets. That calls for a very liberal business environment, where entrepreneurs will test and try a variety of ideas. Artificial interference is highly likely to not yield any positive results. However, the greater economic output is most likely achieved where the business sector cooperates with science in the areas where competitive advantage has already been established.

3.3. The State: An Insurance Policy for the Average Citizen

In the previous subsection, the diversification of risks was discussed horizontally, i.e. across different sectors or areas of activity. It is important to note, that diversification is also very important vertically, i.e. within a single area. The complexity of a business model (or produced goods and services) is of course a function of its placement in the Lancasterian evolution tree. As the empirical evidence suggests, the power law distribution also applies to firm sizes of the companies (Axtell, 2001; Gabaix, 2016). Therefore, it is highly likely that only a relatively small part of the population will be employed in industry leading companies and perhaps the dominant part of the population will be employed in mediocre, lower value added companies. The power laws in income distribution (Takayasu, 2013 and references therein) and the socio-economic construction of the society implies that the majority of the employees receive less than the average income. The opinion that the economy should be distributed by skill level is influential in poverty reduction – countries like Estonia need to maximise their employment rate in order to avoid emigration and therefore a further decline in population size.

Reducing poverty and increasing prosperity are diametrically opposed strategies from an ideological viewpoint. Karl Popper has argued that a person cannot be genuinely happy for the success of another person; however, s/he can be genuinely sorry for the suffering of another person (Popper, 1945). Given the inequalities in income (and power laws in income distribution), there is no point in speaking about increased prosperity as the goal, as the average is influenced predominantly by upper deciles – the elite. The situation appears to be asymmetrical: subtracting the minus produces a greater amount of well-being than adding the plus. An important aspect to bear in mind is that the reduction of poverty must not be regarded as securing a growth in social benefits, but should be aimed at boosting wealth production in the society. The same principle is conveyed by a well-known metaphor – teaching a man to fish is a far better idea than simply giving him a fish.

A reasonable welfare state together with its institutions has to ensure an adequate standard of living for an ordinary person, including decent work, a living wage and human well-being. To achieve maximum employment, jobs are required at all skill levels. This is especially important for countries like Estonia that have small and declining populations, which are, at the same time, the carrier of the language and culture. If job creation falls under a critical level, the citizens start to emigrate, looking for better economic conditions. Mass emigration is very dangerous from a cultural perspective, as it directly endangers the culture and language. To conclude, from a national perspective, job creation at all income levels must be emphasised. If the jobs are not present in a variety of specialisation levels, the lack of employment opportunities can be a direct threat to culture and language.

3.4. A Complex Endogenous Growth Model for Estonia

Throughout this paper it has been highlighted that the global economy, concerning business as well as science, is becoming increasingly complex. New industries with new companies are born all the time, replacing existing ones. The same tendency occurs in science. This phenomenon has been discussed in the context of the endogenous growth model, i.e. a growth model where the growth part is not exogenous, but comes from the economy itself. Moving up in complexity has been characterised using the Lancasterian evolution tree, embedding bifurcations along its paths. In the paper it has been suggested that the bifurcations are positively related with the scalar product of vectors describing structures of science and economy in the country. The conclusion was straightforward – the economic output is maximised, when the structures of science and economy overlap.

When considering R&D investments, the main conclusion drawn in this paper is that closer correspondence between business and R&D results in a higher output for the country. In other words, it does not matter what the amount of research funding is compared to GDP, what matters more is how the science funding can be capitalised on by the business sector. Science funding should not be treated merely as a budgetary decision by governments – R&D funding is, by essence, an investment and investment decisions should be made based on revenue streams. It is very important to note that this is not absolute! There are plenty of scientific areas where commercial profitability is not relevant. And similarly, there are plenty of business initiatives that do not require any scientific innovation. However, it must always be remembered that economic output is positively related with the overlap of science and economy.

The main goal of the Republic of Estonia is to preserve its native language and culture. The greatest threat to the survival of our country is presented by population decline and outward migration of our own people. It is suggested, that the state must, at the same time, be an insurance policy for its people, so as to preserve the number of tradition bearers. These tasks involve a triple challenge:

- Enabling an adequate standard of living (including employment) for citizens – which is achieved through job creation in all levels of specialisation;
- Creating an environment for positive development leaps in business, R&D and preferably in their concurrence (regarding economic effectiveness as the product of R&D and industry);
- Ensuring the preservation and reproduction of the culture and language.

4. Conclusion and Discussion for Further Research

In this paper, the endogenous growth model was discussed and motivated within the existence of complex systems. Also, applications were given to devise policies in order to maximise the output of Estonia.

Future research, based on the topics discussed in this paper, can be pursued in many areas. As a first step, a proper mathematical formulation of the growth parameter A(t) embedding bifurcations should be devised. After that, the endogenous growth model can really be taken to the next level, corresponding to the empirical evidence of economic development. It would be also very important to provide empirical tests for an altered Romer growth model. This, of course, can be done even without the precise mathematical formulation of bifurcations. Further, the idea postulated that economic output is a scalar product of the vectors of structures of science and economies should be tested with multiple time periods as well as in multiple countries.

Finally, as a separate discussion, job creation on various specialisation levels should be addressed. Complex systems, such as self-organising systems, may yield opposite outcomes, i.e. regions or even the whole country may be left without any competitive advantages. Therefore, it is beneficial to discuss the mitigation of such risks and how to achieve job creation in conditions of limited business or research opportunities and also where there is no competitive advantage.

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