Testing for Rational Speculative Bubbles on the Estonian Stock Market [1]

Dmitry Kulikov
Bank of Estonia
Estonia pst. 13, 15095 Tallinn, Estonia
Phone: +372 6680777, e-mail, dmitry.kulikov@eestipank.ee

Abstract

This paper looks for empirical evidence of rational speculative bubbles on the Estonian stock market during the period 1996–1999. Four different testing methodologies are used in the paper: two tests are indirect, looking for statistical "footprints" of the speculative dynamics in the data, and two other tests are based directly on testable implications of the Blanchard and Watson (1982) model of periodically collapsing rational speculative bubbles. The paper finds tentative support for the bubble hypothesis on the Estonian stock market during the period that includes the Asian and the Russian financial crises of 1997 and 1998. The paper offers an empirical contribution to the literature on financial markets efficiency in the emerging Central and Eastern European economies.

JEL classification codes: G12, G14, C58
Keywords: rational speculative bubbles, regime-switching models, stock market prices

[1] The views expressed in this paper are those of the author and do not necessarily represent the official position of Eesti Pank. I am grateful to the seminal participants at the Center for European Integration Studies, Bonn for their helpful comments. All remaining errors and omissions are my own.

1. Introduction

Since the beginning of the 1990s Central and Eastern European nations have experienced rapid political and economic changes, giving rise to the increase of living standards and economic opportunities. On the way to prosperity the creation of viable and stable economic institutions is vital. Banking and financial markets play an especially prominent role, facilitating and speeding up economic development, but at the same time introducing an added dimension of volatility and risks. This paper takes a look at the relatively recent history of the Estonian stock market through the prism of financial speculation and asset price bubbles.[1]

The Tallinn Stock Exchange was established in the first half of 1995, with the actual trading of financial securities starting June 1996.[2] Though comparatively small by both the capitalisation and the number of traded assets, the official stock market index (TALSE) had experienced dramatic fluctuations during the period 1996 to 1999. Starting off from the initial value of 100 in June 1996, TALSE had climbed up to nearly 500 by September 1997, before falling sharply to around 350 in the aftermath of the Asian financial crisis and then declining steadily back to the starting position of around 100 in October 1998 after the Russian financial crisis. Another noteworthy feature of the market since the crash in October 1997 had been an elevated level of volatility; see Figure 1. This stylised picture of the Estonian stock market in 1996–1999 may suggest that it had been susceptible to market speculation during that period. Its small size, limited number of participants and a narrow range of traded securities informally support this hypothesis.

This paper tests for rational speculative bubbles on the Estonian stock market during the time period from June 1996 to March 1999. As usual in the empirical studies of this kind, the findings of this paper need to be interpreted with a measure of caution. Firstly, just one particular type of the rational speculative bubble, due to Blanchard and Watson (1982), is tested for. In addition, other bubble-like explosive stochastic processes in the sample are probed using a battery of indirect statistical tests. However, the findings in this paper may only suggest the presence of certain data features that are consistent with some particular models of the speculative dynamics, and do not offer a comprehensive answer to the bubble hypothesis.

Secondly, empirical rational speculative bubbles' tests usually require correctly specified market fundamentals, which are mostly derived from the history of dividend payouts. However, the Estonian stock market during the period 1996-1999 had no available dividend history and no other empirically suitable proxy for market fundamentals. In order to

[1] This paper was written in the second half of 1999, almost one decade before the recent economic down- turn, precipitated by the collapse of Lehman Brothers and ensuing financial market panic in fall 2008. As documented in Reinhart and Rogoff (2009) and many other recent papers, financial crises are not unusual in the recent economic history of both developing and developed nations alike. The focus of this paper is on the Estonian stock market collapse in the fall of 1997, in the aftermath of the Asian financial crisis. In retrospect, these distant events completely reshaped the banking sector in Estonia, helping to forge a solid base for the next decade's robust economic growth. Likewise, the most recent events have already reshaped the perception of the financial markets and their complex links to the economic development among economists, politicians and the general public alike. The lessons are being learned and changes are taking place, their effects will be felt in the decades ahead.

[2] A short historical account of the Tallinn Stock Exchange can be found at the NASDAQ OMX Baltic website www.nasdaqomxbaltic.com

overcome this limitation, the paper applies two bubble tests that do not rely on market fundamentals, instead looking for certain statistical "footprints" left by the speculative market dynamics. Again, any interpretation of the empirical findings in this paper has to bear that in mind.

This paper makes use of the following direct and indirect bubble testing methods. The first test, due to Engel and Hamilton (1990), is essentially a Markov switching model of stock market returns. This indirect test enables to see if TALSE is characterised by different regimes and to study empirical properties of the regime switching process. The second indirect test is based on Hall, Psaradakis and Sola (1999) Markov switching ADF regression that enables looking for possible explosive roots in the stock market prices. The third bubble test makes use of the McQueen and Thorley (1994) approach, looking for a specific form of duration dependence in the same-sign runs of stock market returns. In addition to the original form of the duration dependence test, an extension of this approach encompassing a broader set of bubble processes is also proposed in the paper. Finally, several types of the i.i.d. switching regression models based on van Norden and Schaller (1993) study of rational speculative bubbles are also applied to the Estonian stock market data.

To the author's best knowledge, this paper is one of the few empirical studies addressing the issue of rational speculative bubbles on the newly created stock markets in Central and Eastern Europe. The previous literature that covers the second half of the 1990s is very limited. Among the relevant references are financial market information efficiency studies by Zalewska- Mitura and Hall (1998), Hall, Urga and Zalewska-Mitura (1998) and Korhonen (1998), as well as the paper by Shields (1997) that investigates stock market volatility in the emerging Central and Eastern European economies. The market efficiency studies tend to conclude that the newly created financial markets exhibit a weak-form inefficiency, gradually becoming more efficient over time. The rational speculative bubbles may be viewed as another form of the market inefficiency, where prices tend to deviate from their long run (non-bubble) equilibrium level, effectively diverting investors' resources from the efficient use. One of the contributions of this paper, therefore, is to the body of empirical market efficiency studies on transition economies.

The paper is organised as follows. Section 2 examines theoretical issues related to the rational speculative bubbles in asset prices. In particular, a generalised version of the Blanchard and Watson (1982) bubble specification, that serves as a template for the two empirical bubble tests in this paper, is inspected in some detail. The empirical tests are presented in Section 3, which goes step-by-step from the theoretical foundations and their implications to the specific likelihood function form for each particular test. Section 4 overviews the Estonian stock market data and gives some interpretations of the events during the time period 1996-1999. Section 5 presents the empirical findings for each individual test in turn. Finally, the conclusion summarises all empirical results and discusses some avenues for future research.

2. Rational Speculative Bubbles

Rational speculative bubbles have a long history in economics. John Maynard Keynes (1936) described equity markets as an environment where speculators anticipate "what average opinion expects another average opinion to be", rather than focusing on things fundamental to the market. Starting from the seminal study by Robert Shiller (1981), the issue of rational speculative bubbles on the US stock market has generated a large amount of literature; refer to Blanchard and Watson (1982), Tirole (1982, 1985), Diba and Grossman (1987), Kenneth West (1987) among others. Two surveys that cover both theoretical and empirical aspects of rational speculative bubbles include Camerer (1989) and Flood and Hodrick (1990).

The rational speculative bubble is a permanent or temporary deviation of asset prices from their "fundamental" value, and is the result of self-fulfilling market dynamics. The bubble is rational, because from each individual market participant point of view it does not lead to expected losses nor profits. Instead, participation in the bubble leads to redistribution of the market wealth across the participants, as some realise large profits, while others suffer losses. It is the large potential profit component (adjusted for the risk) that attracts participation in the rational speculative bubble.

Formally, let an asset price p_t be given by:

$$p_t = f(\boldsymbol{x}_t) + \beta \mathrm{E}_t p_{t+1}, \tag{1}$$

where E_t denotes the conditional expectation operator based on the information set at time t, $0 < \beta < 1$ is the time preference parameter, and $\boldsymbol{x}_t$ is a vector of economic "fundamentals" for the particular asset. Iterating on this equation for T periods forward:

$$p_t = \sum_{j=0}^{T-1} \beta^j \mathrm{E}_t f(\boldsymbol{x}_{t+j}) + \beta^T \mathrm{E}_t \mathrm{p}_{t+T} .$$

When the transversality condition $\lim_{T\to\infty} \beta^T \mathrm{E}_t p_{t+T} = 0$, is imposed, the resulting fundamental price, denoted p^f, is given by:

$$p^f_t = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t f(\boldsymbol{x}_{t+j}) .$$

The fundamental price is therefore determined only by the underlying economic factors that affect the price of a particular asset.

However, p^f_t is not the only solution of (1). If the transversality condition is omitted, a whole new range of possible solutions arise, each having the following affine structure:

$$p_t = p^f_t + b_t, \tag{2}$$

where b_t denotes a "non-fundamental" bubble term. Since b_t needs to satisfy (1), each bubble trajectory is characterised by:

$$\mathrm{E}_t b_{t+1} = \frac{1}{\beta} b_t . \tag{3}$$

Therefore, once initiated, the "non-fundamental" bubble component grows at the rate of $\frac{1}{\beta}$ per period. These types of bubbles are referred to as explosive bubbles, because they are supposed to continue growing forever after inception.

In the real world, bubbles are not likely to inflate (or deflate) asset prices forever. Blanchard and Watson (1982) introduce the model of periodically collapsing rational speculative bubbles that still satisfy (3). Their bubbles exist in two different states: the survival state, S, and the collapse state, C:

$$\mathrm{E}_t\, b_{t+1} = q\, \mathrm{E}_t(b_{t+1} \mid S) + (1-q)\, \mathrm{E}_t(b_{t+1} \mid C)\ , \tag{4}$$

where $0 \le q \le 1$ denotes the bubble survival probability in each time period.[3] Let us for now assume that in the collapse regime bubbles disappear completely:

$$\mathrm{E}_t(b_{t+1} \mid C) = 0\ . \tag{5}$$

Combining equations (3) and (4) leads to the bubble trajectory in the survival regime:

$$\mathrm{E}_t(b_{t+1} \mid S) = \frac{1}{\beta q}\, b_t\ . \tag{6}$$

It transpires that, in the survival mode, the rational speculative bubbles are expected to grow at the rate $\frac{1}{\beta q} \ge \frac{1}{\beta}$ in order to compensate investors for the risk of the future collapse. The Blanchard and Watson (1982) model can further be generalised to allow for time-varying probability of survival and for gradual "deflation" of bubbles over several time periods in the collapse regime. It might be argued, that the bubble survival probability in each time period is inversely proportional to the current bubble size:

$$q = q(b_t)\ , \frac{dq(b_t)}{db_t} < 0\ . \tag{7}$$

Similarly, gradual "deflation" of bubbles in the collapse regime is given by the following generalisation of (5):

$$\mathrm{E}_t(b_{t+1} \mid C) = u(b_t)\ .$$

Assuming that $u(b_t)$ is a continuous everywhere differentiable function that satisfies:

$$u(0) = 0\ , \qquad 0 \le \frac{du(b_t)}{db_t} \le 1\ , \tag{8}$$

the bubble is expected to shrink in the collapse regime, i.e. $|u(b_t)| \le |b_t|$. Taken together, the following general specification of Blanchard and Watson (1982) periodically collapsing rational speculative bubbles is used in Section 3 to derive testable implications of the bubble hypothesis:

$$\mathrm{E}_t\, b_{t+1} = \begin{cases} \dfrac{1}{\beta q(b_t)}\, b_t - \dfrac{1-q(b_t)}{q(b_t)}\, u(b_t) & \text{with probability } q(b_t) \\ u(b_t) & \text{with probability } 1 - q(b_t)\ . \end{cases} \tag{9}$$

The theory of rational speculative bubbles had been a subject of criticism. In a representative agent model, the rational agent is not expected to buy an over-valued asset, thus preventing inception of bubbles and thereby their very existence. Another argument by Diba and Grossman (1987) is that bubbles cannot grow negative, since modern financial markets are based on the limited liability principle. However, equation (3) is symmetric,

[3] The bubble survival probability for $n \ge 0$ periods equals q_n; its expected duration is given by $\frac{1}{1-q}$.

meaning that b_t can potentially be negative and over time push asset prices into negative territory.[4] On the other hand, explosive positive rational speculative bubbles are expected to reach market capitalisation limits in the finite number of time period after their inception – something that is never observed in reality.

While many critics rule out asset price bubbles, their arguments rely on the rational expectations' assumption. Blanchard and Fischer (1989), on the other hand, argue that, "[Theoretical restrictions] often rely on an extreme form of rationality, and are not, for this reason, altogether convincing. Often bubbles are ruled out because they imply, with a very small probability and very far in the future, some violation of rationality, such as non-negativity of prices or the bubbles becoming larger than the economy. It is conceivable that the probability may be so small, or the future so distant, that it is simply ignored by market participants."

From the point of view of this paper, a more pertinent issue concerns possible alternative explanations of the "bubble-like" trajectories in asset prices. As suggested, among others, in Flood and Hodrick (1990), the bubble model is always observationally equivalent to the no-bubble model with differently specified market fundamentals. For example, a number of recent studies have entertained the possibility that anticipated future changes in market fundamentals may lead to episodes reminiscent of the asset price bubbles; see Flood and Hodrick (1986) and van Norden and Schaller (2002). Therefore, any empirical evidence of bubbles in the real world data must always be given a benefit of the doubt, because different specifications of market fundamentals may lead to a completely opposite empirical result. However, in many cases, the economic theory is sufficiently unambiguous about the list of proxy variables suitable for the market fundamentals in each particular asset category.

3. Four Empirical Tests for Rational Speculative Bubbles

This section presents four tests of the rational speculative bubbles that are used in the empirical part of this paper. Two of the tests in subsection 3.1. are indirect: they do not depend on a specific bubble model and are designed to look for generic "footprints" of speculative dynamics in the data. The other two tests, in subsections 3.2. and 3.3., are derived directly from the Blanchard and Watson (1982) model of the periodically collapsing rational speculative bubbles, and can therefore be regarded as direct bubble tests.

3.1. Markov Switching Models of Stock Market Dynamics

Without appealing to any particular theoretical bubble framework, the speculative dynamics has the following "typical features": in one regime, asset prices steadily grow and market returns are positive with a relatively low volatility; in the other regime, asset prices go down rapidly and market returns are large and negative. Asset returns in such markets are characterised by autocorrelation, negative skewness and excess kurtosis. In addition, as noted in Schwert (1990), in the post-crash markets the volatility of returns often remain elevated for long periods of time. Statistical tests of these "footprints" can be accomplished

[4] Weil (1990) discusses the possibility of negative bubbles in asset prices.

using the class of hidden Markov models, where in each time period the statistical model of asset prices depends on an unobserved state of the underlying discrete Markov process.[5]

This subsection introduces two indirect tests that make use of the two-state hidden Markov model to look for generic statistical "footprints" left by the speculative bubbles. The first is the mixture of normals model by Engel and Hamilton (1990), previously used by Schwert (1990) and Ahmed, Rosser and Uppal (1996) to test for bubbles on the US and Pakistani stock markets respectively. The second test is due to Hall, Psaradakis and Sola (1999); it relies on the switching ADF-type regression for uncovering explosive roots in the stock market data.

Let a hidden state variable $s_t \in \{ S, C \}$ indicate the market regime in each time period t. In the Engel and Hamilton (1990) mixture of normals model, market returns $r_t := \frac{p_t}{p_t-1} - 1$ are drawn from the following state-dependent normal distribution:

$$r_t \mid s_t \sim N(\mu_{s_t}, \sigma^2_{s_t}) . \tag{10}$$

The bubble growth regime corresponds to $\mu_S > 0$, while the collapse regime with elevated volatility amounts to $\mu_C < 0$ and $\sigma^2{}_S < \sigma^2{}_C$.

In the switching ADF test of Hall, Psaradakis and Sola (1999) the following state-dependent model of stock prices is postulated:

$$\Delta p_t = \mu_{s_t} + \phi_{s_t} p_{t-1} + \sum_{j=1}^{k} \psi_{s_t j} \Delta p_{t-j} + \varepsilon_t , \tag{11}$$

where ε_t is an independent $N(0, \sigma^2)$ innovation in both regimes, and $k \geq 0$ is the augmentation length. This test is a generalisation of the Dickey and Fuller (1981) regression, modified for detecting explosive autoregressive roots $\phi_S > 0$ in the bubble growth regime. The idea of the switching ADF test is therefore to separate the random walk and the explosive periods in the sample of asset prices p_t, letting the data decide if and when the process switches between the two regimes.

Let the hidden state s_t evolve according to a two-state discrete Markov chain, governed by the 2×2 transition matrix $\mathbf{P}$. This simple specification is sufficient to characterise recurrent processes that mostly stay in one of the regimes, when, say, the first main diagonal element, p_{11}, of the transition matrix $\mathbf{P}$ is close to one, with only brief visits to the other regime, when the second main diagonal element, p_{22}, is close to zero. This corresponds to prolonged run-ups in asset prices in the bubble growth phase, followed by short and abrupt collapses.

Let $\boldsymbol{\xi}_{t|t} := \mathrm{P}(s_t | F_t; \boldsymbol{\theta})$ denote the conditional distribution of s_t based on the current information set F_t, where the vector of model parameter is denoted $\boldsymbol{\theta}$. Let the state probability for the first observation in the dataset be denoted by $\boldsymbol{\rho} := \boldsymbol{\xi}_{1|0}$, and let it be estimated together with θ. Then the optimal inference about s_t can be found by iterating on the following system of equations for $t = 1, \dots , T$:

$$\boldsymbol{\xi}_{t|t} = \frac{\boldsymbol{\xi}_{t|t-1} \circ \eta_t}{\mathbf{1}^{\mathrm{T}}(\boldsymbol{\xi}_{t|t-1} \circ \boldsymbol{\eta}_t)}, \quad \boldsymbol{\xi}_{t+1|t} = \mathbf{P}\, \boldsymbol{\xi}_{t|t} , \tag{12}$$

where o denotes Hadamard (element-by-element) product operator, **1** is a conformable

[5] One of the first applications of hidden Markov models in econometrics is by Goldfeld and Quandt (1973). More recently, availability of high-performance computers rekindled the interest in regime-switching time series models after the seminal work of Hamilton (1989). For a textbook treatment of the hidden Markov models refer to Cappé, Moulines and Rydén (2005).

vector of ones, and $\boldsymbol{\eta}_t$ denotes the vector of state-dependent likelihoods for observation t. For the Engel and Hamilton (1990) mixture of normals model, $\boldsymbol{\eta}_t$ consists of two normal probability densities evaluated at r_t according to (10). For the Hall, Psaradakis and Sola (1999) switching ADF regression, $\boldsymbol{\eta}_t$ contains linear regression model likelihoods for each of the two states according to (11). Finally, the log likelihood function $L(\boldsymbol{\theta},\boldsymbol{\rho}; \{p_t : 1 \leq t \leq T\})$ for the observed sample of market prices $\{p_t : 1 \leq t \leq T\}$ is given by:[6]

$$L_T(\boldsymbol{\theta},\boldsymbol{\rho}; \{p_t : 1 \leq t \leq T\}) = \sum_{t=1}^{T} \log 1^{\mathrm{T}}(\xi_{t|t-1} \circ \eta_t).$$

3.2. Runs Duration Dependence Tests

As pointed out in Subsection 3.1., the speculative dynamics in asset markets leads to prolonged run-ups of prices, followed by large and abrupt losses, suggesting the use of runs duration test to look for bubbles. Blanchard and Watson (1982) apply this idea to the gold market, failing to reject the null hypothesis of no bubbles. However, as they point out, autocorrelation in asset returns can arise from factors other than rational speculative bubbles. The long-run returns autocorrelation can be induced by fads as in Poterba and Summers (1988), or stem from the time-varying risk premium as in Fama and French (1988). In the series of daily or weekly returns, autocorrelation is a well-documented fact, explained by the non-synchronous trading and calendar effects. McQueen and Thorley (1994) suggest a more discriminating type of the runs duration test, capable of telling apart bubble-induced runs durations from other types of autocorrelation in asset returns.

Let an asset price innovation $\tilde{r}_t$, also known as abnormal return, be a sum of two components: the "fundamental" innovation ε^f_t and the "bubble" innovation ε^b_t. From (1) and (3), the two innovations are defined as:

$$\varepsilon^f_t := p^f_t - \frac{1}{a}[p^f_{t+1} - f(\boldsymbol{x}_t)], \qquad \varepsilon^b_t := b_t - \frac{1}{a} b_{t-1}.$$

Let the innovation ε^f_t have normal distribution around zero, corresponding to the efficient market hypothesis, where the fundamental price is a random walk. The bubble innovation, on the other hand, is assumed to switch between the survival and collapse regimes, as in (4). This assumption directly links ε^b_t to the Blanchard and Watson (1982) periodically collapsing bubble dynamics in equation (9).

Next, consider the probability that $\tilde{r}_t$ is negative, $\mathrm{P}(\tilde{r}_t < 0)$. McQueen and Thorley (1994) assume that the time-invariant probability of collapse satisfies $1-q(b_t) \equiv 1-q < \frac{1}{2}$, and that bubbles fully collapse in one time period, i.e. $u(b_t) \equiv 0$. Using these simplifying assumptions, they derive the following testable implication of the Blanchard and Watson (1982) bubble model:

$$\frac{d\mathrm{P}(\tilde{r}_t < 0)}{db_t} < 0. \tag{13}$$

[6] The parameter estimates can be obtained using the expectation-maximisation (EM) algorithm, since the direct numerical maximisation of this log likelihood is too complicated. Refer to Hartley (1978), Hamilton (1990) and Kim (1994) for further details on the EM algorithm.

McQueen and Thorley (1994) further make an important assumption that the unobserved bubble size b_t can be inferred from the same-sign interval (run) durations of $\tilde{r}_t$. This assumption enables statistical testing of the implication (13) using the tools of duration data modelling.[7] In particular, the declining probability of observing a negative abnormal return in the bubble growth regime implies a negative duration dependence in the runs of positive $\tilde{r}_t$-s:

$$h(k) := P(\tilde{r}_t < 0 \mid \tilde{r}_{t-1} > 0, \ldots, \tilde{r}_{t-k} > 0, \tilde{r}_{t-k-1} < 0), \quad h(k) \geq h(m) > 0,$$

where h denotes the hazard function and $0 \leq k \leq m < \infty$ its integer run length argument.

An extension of the McQueen and Thorley (1994) runs duration dependence test is proposed in this paper. In contrast to the former, the extended test relaxes the simplifying assumptions of time-invariant collapse probability and full bubble deflation in one period, and therefore corresponds to the most general form of the Blanchard and Watson (1982) periodically collapsing bubble model.[8] A summary of the main implications of this extended runs duration dependence test is given below:

- The partial bubble collapse extension, $u(b_t)$, does not affect the basic proposition of the negative duration dependence in the same-sign runs of $\tilde{r}_t$. However, it makes the hazard function flatter compared to the baseline McQueen and Thorley (1994) case. In practice this may lead to higher rejection rates of the null hypothesis, since the negative duration dependence is less pronounced. On the other hand, partial bubble collapses lead to longer observed runs of the same-sign $\tilde{r}_t$-s, since the probability of large negative innovations in the collapse regime is smaller;
- The bubble size-dependent collapse probability, $1-q(b_t)$, leads to the non-monotonic duration dependence in the runs of same-sign $\tilde{r}_t$-s. Theoretically the most plausible scenario is for the hazard function to decline initially, while the bubble size is still small, but to start increasing again after b_t exceeds a certain "critical mass". This corresponds to $1-q(b_t) < \frac{1}{2}$ when the bubble is small, monotonically approaching unity as b_t grows. Other kinds of duration dependence are less conceivable from the theoretical perspective.

The following hazard function for the extended runs duration dependence test is proposed in this paper:

$$h(k) = \frac{1}{1 + \exp^{-(\alpha+\beta k+\gamma k^2)}} . \tag{14}$$

This parameterisation allows for various forms of duration dependence that are summarised in Table 1, nesting the baseline McQueen and Thorley (1994) case when $\gamma = 0$. The maximum likelihood estimation of the extended model uses the following log likelihood function:

$$L_N(\boldsymbol{\theta}; \{k_i : 1 \leq i \leq N\}) = \sum_{i=1}^{N} \log \left[h(k_i) \prod_{m=1}^{k_i-1} (1-h(m))\right],$$

[7] Analysis of duration data has a large body of econometric literature, with application in labor economics, unemployment studies, and high-frequency financial data econometrics. See Lancaster (1990) for an in-depth survey of the duration analysis literature and methods.

[8] Mathematical appendix with full derivation details of the extended runs duration dependence test can be found in the unabridged version of this paper, which is available on request from the author.

where the sample of same-sign run durations of abnormal returns is denoted $\{k_i : 1 \leq i \leq N\}$, and $\boldsymbol{\theta} = (\alpha, \beta, \gamma)^{\mathsf{T}}$. Maximisation of this log likelihood can be done by the usual numerical optimisation routines, with the asymptotic variance-covariance matrix estimate found as the negative inverse Hessian evaluated at the function's maximum.

Table 1. Duration Dependence in the Extended Test

Parameters	Duration dependence
$\beta < 0$, $\gamma > 0$	U-shaped hazard function; inflection at $-\frac{\beta}{\gamma}$
$\beta > 0$, $\gamma < 0$	∩-shaped hazard function; inflection at $-\frac{\beta}{\gamma}$
$\beta < 0$, $\gamma = 0$	Monotonically decreasing hazard function
$\beta > 0$, $\gamma = 0$	Monotonically increasing hazard function

Source: Author's compilation

A significant advantage of the duration dependence tests discussed in this subsection is their independence from the market fundamentals. In contrast to the i.i.d. switching regression test in subsection 3.3., the methodology of McQueen and Thorley (1994) is robust to misspecifications of p^f_t, which is particularly important in the case of TALSE data. On the other hand, the tests in this subsection critically depend on the kind of data used to compute run lengths, which serve as a proxy for the unobserved bubble size b_t. Daily abnormal returns have plenty of idiosyncratic noise arising from the day-to-day stock market trading process, and therefore may give misleading inference about the bubble size.[9] One way to overcome this issue is to use weekly or monthly returns instead, but in the case of Estonian stock market data this approach substantially reduces the available sample information. The middle ground path pursued in this paper is to use both daily and weekly data samples for the runs duration dependence testing in Section 5.

3.3. I.i.d. Switching Regression Tests

Another direct test for rational speculative bubbles is proposed in van Norden and Schaller (1993) and van Norden (1996). Based on the observation in McQueen and Thorley (1994) that periodically collapsing bubbles lead to a distinct regime switching behaviour in abnormal returns, they suggest explicit modelling of this process using an i.i.d. switching regression approach. The bubble size b_t, needed for this test, is found directly from (2). The fundamental price, p^f_t, is therefore a critical part of the van Norden (1996) testing methodology, and all empirical findings are conditional on the particular specification of market fundamentals.[10]

Recall from subsection 3.2. that $\tilde{r}_t = \varepsilon^f_t + \varepsilon^b_t$. Unconditional expectation of $\tilde{r}_t$ is zero, because $E\varepsilon^f_t = 0$ by assumption and $E\,\varepsilon^b_t = 0$ from equation (3). On the other hand, conditional of the bubble regime, the expectations of ε^b_t and $\tilde{r}_t$ are no longer zero; see (9). As explained

[9] See Chan, McQueen and Thorley (1998) on the issue of high-frequency data in the duration dependence tests.

[10] Usually in the applied studies, history of dividend payouts is considered to be a good proxy for the market fundamentals. In the case of TALSE, however, this kind of data is not available. In an application to Pakistani stock market, Ahmed, Rosser and Uppal (1996) face similar limitations, relying instead on daily exchange rate series for a market fundamentals' proxy. Due to the currency board system in place during the sample period, their approach is not feasible for the Estonian stock market.

in van Norden and Schaller (1993) and van Norden (1996), the Blanchard and Watson (1982) model of periodically collapsing bubbles implies the following testable sign restrictions:

$$\frac{dE_t(\tilde{r}_{t+1}|\,S)}{db_t} = \frac{-q'(b_t)}{q(b_t)^2}\left[\frac{1}{a}\,b_t - u(b_t)\right] + \frac{1-q(b_t)}{q(b_t)}\left[\frac{1}{a} - u'(b_t)\right] > 0,$$

$$\frac{dE_t(\tilde{r}_{t+1}|\,C)}{db_t} = -\left[\frac{1}{a} - u'(b_t)\right] < 0.$$

Intuitively, the first restriction is due to the exponential bubble growth in the survival regime, where each ε^b_t is a linear function of the b_{t-1}; see (6). The negative sign in the second arises from the fact that the bigger is b_{t-1}, the larger ε^b_t is needed in the collapse regime for the bubble to shrink.

These two restrictions can be tested in a simple framework of the i.i.d. switching regression model. Linearising $E_t(\tilde{r}_{t+1}|\,S)$ and $E_t(\tilde{r}_{t+1}|\,C)$ around zero and parameterising appropriately, the following state-dependent linear model can be used to test validity of the bubble hypothesis:

$$\tilde{r}_t = \gamma_{s_t,0} + \gamma_{s_t,1}\, b_{t-1} + \varepsilon_t\,, \tag{15}$$

where $\varepsilon_t \mid s_t \sim N(0, \sigma^2_{s_t})$ is assumed. The i.i.d. switching regression model is closed by specifying an exible probit model for $q(b_t)$:

$$\mathrm{P}(s_t = S) = \Phi(\gamma_{q,0} + \gamma_{q,1}\,\mathrm{b}_{t-1} + \gamma_{q,2}\,\mathrm{b}^2_{t-1})\,, \qquad \mathrm{P}(s_t = C) = 1 - \mathrm{P}(s_t = S)\,,$$

where Φ denotes the standard normal distribution function. When $\gamma_{S,1} > 0$, $\gamma_{C,1} < 0$ and $\gamma_{q,2} < 0$ are confirmed using statistical tests, the data is consistent with the bubble hypothesis.

The log likelihood function for the i.i.d. switching regression test (15) is given by:

$$L_T(\boldsymbol{\theta}; \{(\tilde{r}_t, b_t) : 1 \le t \le T\}) = \sum_{t=1}^{T} \log\left[\,\mathrm{P}(s_t = S)\,\phi(\varepsilon_t; \sigma^2_S) + \mathrm{P}(s_t = C)\,\phi(\varepsilon_t; \sigma^2_C)\,\right],$$

where $\boldsymbol{\theta} = (\gamma_{S,0}, \gamma_{S,1}, \gamma_{C,0}, \gamma_{C,1}, \gamma_{q,0}, \gamma_{q,1}, \sigma^2_S, \sigma^2_C)^{\mathrm{T}}$, and ϕ denotes the normal density function. The EM algorithm is used to obtain the maximum likelihood estimates for this model.

Two special cases of this model are also considered in Section 5. The first is motivated by Cutler, Poterba and Summers' (1991) observation that the mean reversion in asset prices can alternatively be explained by a complex interaction between several different types of traders, where the strict rationality assumption does not apply for some of them. They suggest the following statistical model to test this hypothesis:

$$\tilde{r}_t = \gamma_0 + \gamma_1 b_{t-1} + \varepsilon_t\,, \tag{16}$$

where ε_t is the same as in (15), and the regime switching probability follows Bernoulli law $\mathrm{P}(s_t = S) = \Phi(\gamma_{q,0})$, $\mathrm{P}(s_t = C) = 1 - \Phi(\gamma_{q,0})$.

The second case is a further restriction of the previous model, where market returns are drawn from two distinct volatility regimes, with no connection to asset bubbles:

$$\tilde{r}_t = \gamma_0 + \varepsilon_t\,, \tag{17}$$

where ε_t is the same as in (16).

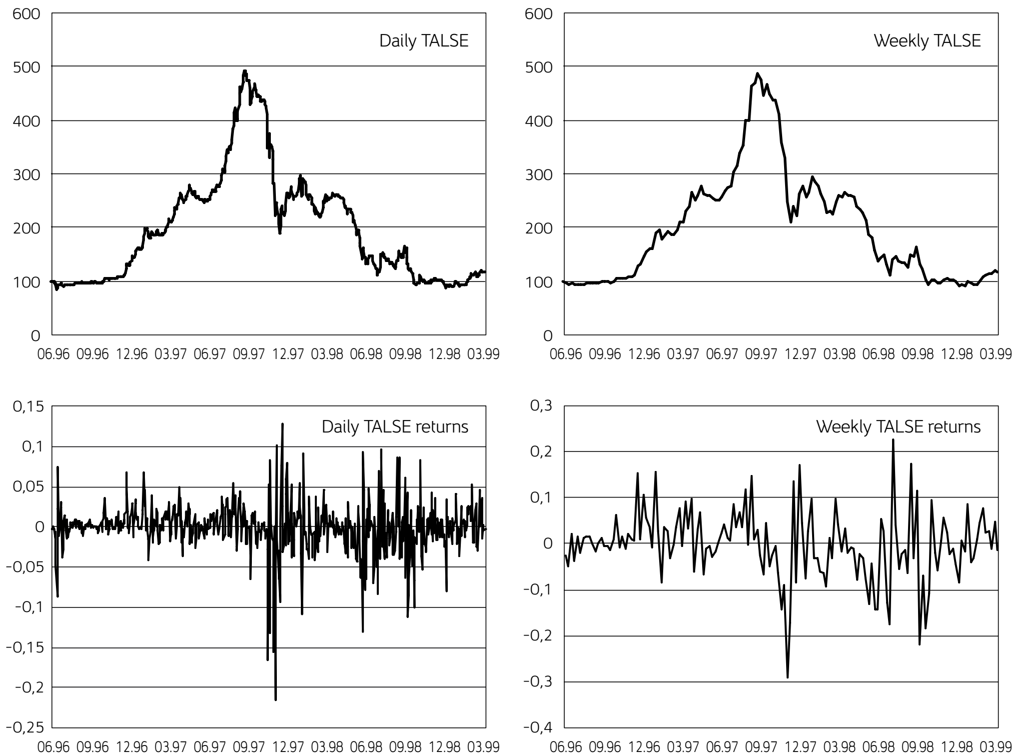
4. Data

This section gives a short description of the Estonian stock market data sample, used in the empirical models in Section 5.

The official Estonian stock market index (TALSE) used in this study, is a capitalisation-weighted index of share prices traded on the two best-quality market lists: the main list and the secondary lists. The number of shares in both lists increased from 11 in June 1996 to 37 by the end of 1998. The TALSE index official base date is 3 June, 1996, when its initial value was set to 100. Throughout the sample period the Tallinn stock exchange electronic quote and order-driven trading platform imposed no restrictions on daily movements of share prices.

The daily and weekly TALSE levels and returns are shown in Figure 1.[11] The daily sample contains 724 observations dated from 3 June, 1996, to 16 March, 1999. The weekly sample is constructed by selecting the index value on Tuesday, Wednesday or Thursday, whichever is available first. This procedure yields 146 observations dated from 5 June, 1996, to 16 March, 1999. As can be seen from Figure 1, weekly data is considerably smoother, but still appears to retain all prominent market features.

Figure 1. Daily and Weekly TALSE Index Levels (Upper Panel) and Returns (Lower Panel)



Source: Author's illustration

[11] Data sample was obtained from the official Tallinn Stock Exchange homepage www.tse.ee.

Table 2 shows descriptive statistics of daily and weekly samples of TALSE returns. The returns volatility on the Estonian stock market differs in the pre- and post-Asian financial crisis sub-samples by a factor of two; see also Figure 1. The statistically significant ARCH effect in the full sample also confirms this empirical regularity. Table 2 reveals that the series is negatively skewed due to several big negative returns in the midst of the crisis.

Substantial excess kurtosis in the daily sample is also present, indicating that returns may come from a mixture of high- and low-volatility regimes. These data features are consistent with the model of periodically collapsing bubbles; refer to the discussion in sections 2 and 3. Based on the significant autocorrelation results in Table 2, the sample of abnormal returns $\{\tilde{r}_t : 1 \leq t \leq T\}$ needed for several tests in Section 5 is computed by filtering out the predictable returns component using an estimated AR(5) linear model. In addition, the usual augmented Dickey-Fuller test of both daily and weekly TALSE index levels confirms the unit root hypothesis, indicating the returns series can be treated as a covariance-stationary process.

Table 2. Descriptive Statistics of Daily and Weekly TALSE Returns

	(a)	(b)	(c)	(a)	(b)	(c)
	Daily returns			Weekly returns		
Mean	0.0002	0.0039	-0.0035	0.0012	0.0197	-0.0172
Std. Deviation	0.0294	0.0170	0.0375	0.0751	0.0495	0.0901
Skewness	-1.1222	0.0041	-0.8414	-0.3258	0.7901	-0.0783
Excess Kurtosis	9.0003	4.4601	5.2887	1.8440	0.8796	0.8039
Minimum	-0.2158	-0.0877	-0.2158	-0.2874	-0.0804	-0.2874
Maximum	0.1287	0.0751	0.1287	0.2306	0.1596	0.2306
Normality	2592.1364 **	299.2200 *	464.6100 *	23.1070 *	9.8116 *	2.0400
Autocorrelation	11.4920 *	14.9310 *	4.4828 *	1.4032	2.2231	0.2644
ARCH	7.5944 *	16.8050 *	2.0708	9.4473 *	2.7418	1.9090
T	723	361	362	145	72	73

Note: Subsamples: (a) June 1996 to March 1999; (b) June 1996 to October 1997; (c) November 1997 to March 1999. Normality test is Jarque and Bera (1987), autocorrelation test is Ljung and Box (1978), ARCH test is Engle (1982). Asterisk denotes significance at the 5% level for the corresponding distribution.

Source: Author's calculations

5. Testing for Bubbles on the Estonian Stock Market

This section presents empirical testing results for rational speculative bubbles on the Estonian stock market between July 1996 and March 1999. Four different testing methodologies, previously covered in Section 3, are applied to the TALSE sample. Subsection 5.1 documents findings of two indirect speculative dynamics tests: although the regime switching behavior is present, it only comprises circumstantial evidence of the rational speculative bubbles on the Estonian stock market. Results of the two direct bubble tests are described in Subsections 5.2. and 5.3. The duration dependence test is found to weakly support the bubble hypothesis: the shape of duration dependence in the runs of positive and negative returns corresponds to the theoretical implications, but few of the statistical results are significant. The i.i.d. switching regression test finds the evidence of bubbles in the case of constant expectations-based fundamentals, but not in the more general case of technical analysis-based proxy for p^f_t.

5.1. Mixture of Normals Model and the Switching ADF Test

Regime switching behavior of stock market prices and returns is documented in many studies; see inter alia van Norden and Schaller (1993), Hamilton and Susmel (1994), McQueen and Thorley (1991), Schwert (1990) and Hall, Psaradakis and Sola (1999). Different regimes in financial data can arise from anticipated switches in market fundamentals, state-dependent heteroscedasticity in the fad component, or indicate the presence of bubbles. This subsection presents empirical evidence of the regime switching behavior in the sample of Estonian stock market data.

Estimation results of the mixture of normals model for both daily and weekly sample of TALSE returns are shown in Table 3. It is seen that in both cases, the second regime is characterised by negative expected return and volatility parameter ten to five times larger than in the first regime. For daily data, however, the mean parameters are not significant at usual levels in either regimes. The evidence of two distinct regimes, one with the average positive return and low volatility, and the other with large negative returns and much higher risk, is e.g. consistent with the stylised description of October 1987 US stock market crash in Schwert (1990).

Table 3. Engle and Hamilton (1990) Mixture of Normals Test

	Daily returns		Weekly returns	
μ_S	0.0011	(0.0007)	0.0144	(0.0054)
μ_S	-0.0039	(0.0041)	-0.0351	(0.0197)
σ^2_S	0.0002	(0.0000)	0.0024	(0.0004)
σ^2_C	0.0025	(0.0007)	0.0127	(0.0035)
p_{11}	0.9733	(0.0055)	0.9762	(0.0175)
p_{22}	0.8967	(0.0287)	0.9362	(0.0488)
L_T	1735.17		190.08	
T	718		146	

Note: Notation for the model parameters corresponds to (10). Main diagonal elements of the transition matrix P are denoted p_{11} and p_{22}. Asymptotic standard errors are shown in the parentheses.
Source: Author's calculations

On the other hand, it is well understood that bubble collapses are relatively short-lived events. In Table 3, however, the expected duration of the collapse regime is 10 days for daily data, and about 16 weeks in the weekly sample.[12] High persistence of the regimes follows from the point estimates of p_{11} and p_{22} elements of the transition matrix **P**. The estimated

[12] This discrepancy is due to the much smoother nature of the weekly sample, seen in Figure 1. High volatility regime in the daily data is likely to be interrupted much more frequently.

probabilities of two regimes are shown in Figure 2, where Kalman filter smoothed $\boldsymbol{\xi}_{t|t}$ are displayed for both the daily and weekly models.

Table 4 demonstrates results of the switching ADF test. The model is estimated on both daily and weekly data using the first difference of the raw TALSE index series as a dependent variable as required by the test methodology.[13] Four lag of augmentation was selected for daily data, and one lag for weekly series.[14] Longer augmentation in daily series is justified by the richer autoregressive structure documented in Table 2. One augmentation lag for the week sample is retained in order to guard against possible non-normality of the disturbance term, since all testing results depend on the asymptotic standard errors.

Table 4. Hall, Psaradakis and Sola (1999) Switching ADF Test

	Daily sample		Weekly sample	
μ_S	-0.1151	(0.3965)	-6.2709	(1.5589)
ϕ_S	0.0016	(0.0018)	0.0837	(0.0156)
μ_C	7.5045	(3.6467)	6.8595	(2.7861)
ϕ_C	-0.0535	(0.0115)	-0.0599	(0.0134)
σ^2	19.9070	(1.1748)	148.4000	(19.5660)
p_{11}	0.9275	(0.0189)	0.7589	(0.1072)
p_{22}	0.2434	(0.0829)	0.8069	(0.0770)
L_T	-2179.23		-586.19	
T	719		144	

Note: Notation for the model parameters corresponds to (11). Augmentation parameters are estimated, but not shown in the table. Main diagonal elements of the transition matrix P are denoted p_{11} and p_{22}. Asymptotic standard errors are shown in the parentheses.
Source: Author's calculations

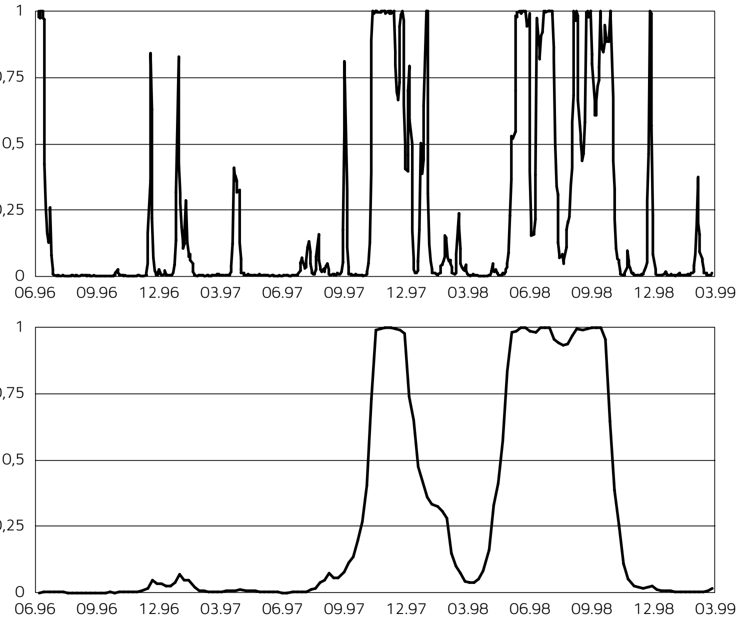
As seen from Table 4, in both samples the switching ADF test discriminates between the two regimes, one of which has an explosive autoregressive root $\phi_S > 0$. However, only in the weekly model this result is statistically significant, while the negative stable root $\phi_C < 0$ is significant in both cases. Note that in both models the point estimate of σ^2 is much higher than in Table 3, which is explained by the lack of the prior log transformation.

Surprisingly, the estimated duration of the bubble growth regime in Table 4 is high relative to the collapse regime in the daily sample. One explanation of this finding may be related to the fact that ϕ_S is not significantly different from zero, meaning that the first regime corresponds to the random walk in stock market prices, while the second regime is stationary. Weekly sample, on the other hand, is consistent with the bubble hypothesis by being a mix of the explosive and stationary regimes. However, these results should be treated as indicative, because hitherto the behavior of market fundamentals remains unknown.

[13] This is a generalised version of the ADF test, where the stationary component of Δp_t is accounted for by the constant term and augmentation. The data is not logarithmically transformed, because this test looks for explosive roots in the data, whereas the log transform may reduce the integration order.

[14] For brevity, these results are not shown in Table 4.

Figure 2. Daily (Upper Panel) and Weekly (Lower Panel) Smoothed Collapse Regime Probability



Source: Author's illustration

5.2. Runs Duration Dependence Test

This subsection looks for the empirical evidence of rational speculative bubbles on the Estonian stock market using the approach of McQueen and Thorley (1994). This testing methodology makes use of the consecutive same-sign runs of stock market returns, looking for a specific form of the duration dependence, implied by the Blanchard and Watson (1982) model of periodically collapsing bubbles; refer to subsection 3.2.

The data sample consists of daily and weekly series of abnormal TALSE returns, see Section 4 for the way $\{\tilde{r}_t : 1 \leq t \leq T\}$ is constructed. The daily sample provides the maximum number of observation, but may contain an added idiosyncratic noise, which is undesirable for this test. The weekly data mitigates the noise issue, but at the cost of substantially smaller sample size. Table 5 gives summary statistics of the constructed runs data along with the non-parametric Kaplan and Meier (1958) hazard function estimates. Both daily and weekly series have approximately equal number of positive and negative runs. This suggests that the potential bubble episodes on the Estonian stock market may not be characterised by abrupt crashes, but rather by piecemeal collapses. McQueen and Thorley (1994) document evidence of longer positive and shorter negative runs duration for monthly US stock market data. In view of this fact, a proposed generalisation of the original McQueen and Thorley (1994) testing methodology that accounts for gradual bubble collapses appears appropriate; refer to Subsection 3.2. for details.

Table 5. Runs Data and the Kaplan-Meier Hazard Function Estimates

	Positive runs			Negative runs		
Length	Runs	KM hazard		Runs	KM hazard	
	Daily sample					
1	79	0.4620	(0.0381)	88	0.5116	(0.0381)
2	40	0.4348	(0.0517)	38	0.4524	(0.0543)
3	29	0.5577	(0.0689)	22	0.4783	(0.0737)
4	7	0.3044	(0.0959)	15	0.6250	(0.0988)
5	3	0.1875	(0.0976)	5	0.5556	(0.1656)
6	8	0.6154	(0.1349)	2	0.5000	(0.2500)
7	3	0.6000	(0.2191)	1	0.5000	(0.3536)
8	1	0.5000	(0.3536)	1	1.0000	–
10	1	1.0000	–			
	Weekly sample					
1	13	0.4333	(0.0905)	18	0.5807	(0.0886)
2	10	0.5882	(0.1194)	5	0.3846	(0.1349)
3	3	0.4286	(0.1870)	1	0.1250	(0.1169)
4	1	0.2500	(0.2165)	3	0.4286	(0.1870)
5	1	0.3333	(0.2722)	2	0.5000	(0.2500)
8	1	0.5000	(0.3536)	1	0.5000	(0.3536)
11				1	1.0000	–
14	1	1.0000	–			

Note: Asymptotic standard errors are shown in the parentheses; refer to Lancaster (1990) for details on the KM estimator and its asymptotic standard errors.
Source: Author's calculations

The Kaplan-Meier (KM) non-parametric hazard function estimates in Table 5 reveal the evidence of U-shaped duration dependence in weekly data, in both negative and positive runs of abnormal returns. This finding is consistent with the bubble hypothesis, but a thorough examination is still required to confirm the statistical significance of this result. On the other hand, there is no pronounced duration dependence in the runs of abnormal daily TALSE returns. The apparent contradiction in the two sets of results may arise due to the noisy daily data.

Table 6 reports results for both the original and extended versions of McQueen and Thorley (1994) duration dependence test using daily and weekly TALSE samples. In almost all cases the shape of the estimated baseline hazard function is consistent with the bubble hypothesis. The original version of McQueen and Thorley (1994) test in weekly returns and in positive runs of daily returns implies a monotonically declining hazard function, because the point estimate of β is negative. Table 6 also reveals that the estimated hazard matches the bubble-consistent U-shape in the extended version of the test for both the daily and weekly samples. However, none of the hazard function shape parameters is statistically significant in daily data: the likelihood ratio test (LR) against the null of constant hazard does not reject the latter, confirming the previous KM non-parametric results in Table 5. In weekly data, on the other hand, the statistically significant duration dependence is found for negative runs of abnormal returns in the extended version of the test. However, this finding may be questioned on the grounds of a relatively small sample size, where the asymptotic standard errors can be misleading. This view is supported by the LR test, which fails to reject the null of no duration dependence.

Table 6. Runs Duration Dependence Test

	Original version				Extended version			
	Positive runs		Negative runs		Positive runs		Negative runs	
	Daily sample							
α	-0.1634	(0.1768)	-0.0966	(0.1991)	0.0143	(0.2920)	0.1576	(0.3517)
β	-0.0081	(0.0649)	0.0557	(0.0861)	-0.1600	(0.2094)	-0.1901	(0.2944)
γ	–		–		0.0212	(0.0279)	0.0418	(0.0484)
L_N	-259.08		-236.84		-258.78		-236.45	
LR	0.0156		0.4204		0.6028		1.2064	
N	171		172		171		172	
	Weekly sample							
α	0.0433	(0.3478)	0.1528	(0.3779)	0.4305	(0.5306)	0.9348	(0.5921)
β	-0.1375	(0.0919)	-0.1605	(0.1131)	-0.4042	(0.2923)	-0.7506	(0.3645)
γ	–		–		0.0229	(0.0232)	0.0635	(0.0366)
L_N	-48.13		-48.09		-47.66		-46.57	
LR	2.6219		2.2410		3.5612		5.2874 *	
N	30		31		30		31	

Note: Notation for the model parameters corresponds to (14). Asymptotic standard errors are shown in the parentheses. The LR test is against the null of constant hazard; asterisk indicates rejection at 10% level.
Source: Author's calculations

As mentioned in Subsection 3.2., one possible explanation for the lack of statistically convincing evidence in Table 5 and 6 may be related to the issue of noisy daily data and its impact on the crucial assumption of the McQueen and Thorley (1994) duration dependence test about the link between the runs length and the bubble size.[15] In the case of weekly data, on the other hand, the signal-to-noise issue becomes less important, while a relatively small number of observations may limit reliability of the asymptotic confidence intervals of standard statistical tests.

[15] In the unabridged version of this paper, the issue of noisy data and its impact on the McQueen and Thorley (1994) duration dependence test is further examined using a Monte Carlo experiment. In particular, the size of McQueen and Thorley (1994) duration dependence test was evaluated for several different parameterisations of Evans (1991) bubble "data generating process" with different levels of the underlying signal-to-noise ratio. The experiment lends support to the conclusion, that the sign of the estimated hazard function shape parameters often gives a more reliable indication of bubbles than the usual 5%-level significance criteria appear to suggest. Further details of the experiment are available on request from the author.

5.3. I.i.d. Switching Regression Test

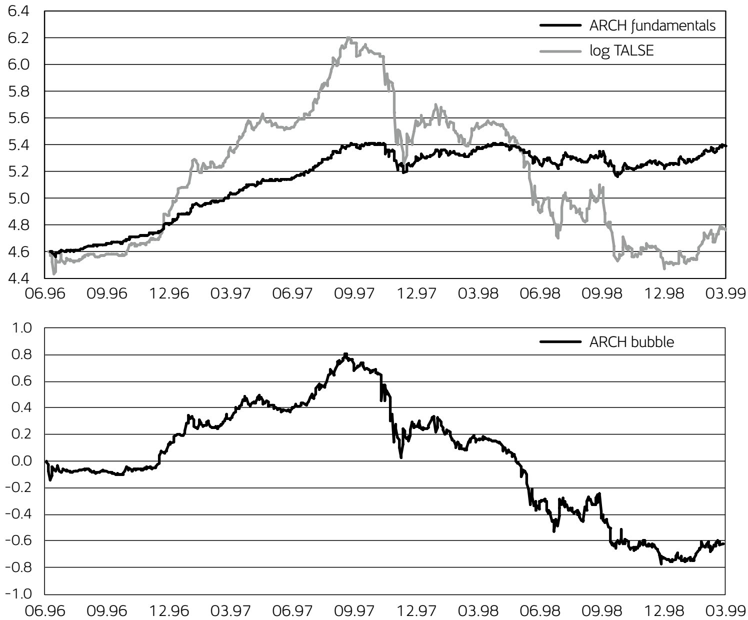
As pointed out in Section 3, the van Norden and Schaller (1993) i.i.d. switching regression test requires an explicit specification of the market fundamentals. In view of the brief market history of stock trading in Estonia, the usual approach of using dividend payouts to construct the fundamentals series is infeasible. Instead, this paper relies on two indirect proxies for the forward-looking expectations of the market participants on the future prospects of the market: one is based on technical analysis-type stock price forecasts, and the other is a simple constant expectations fundamentals. Using these two proxies, the bubble estimate, $\hat{b}_t$, is computed according to (2), where log of TALSE index is used as p_t. The bubble estimate, along with the filtered returns series, is then used to estimate the i.i.d. switching regression model and to test sign and statistical significance of $\gamma_{S,1}$, $\gamma_{C,1}$, $\gamma_{q,1}$, and $\gamma_{q,0}$ parameters to look for the rational speculative bubbles on the Estonian stock market.

In its essence, the technical analysis deals with the next-day stock price forecasts, which is a commonplace practice of market participants. The methods of technical analysis are, however, complex and often rely on large sets of relevant data; refer to Taylor (2005) and references therein. This paper makes use of a relatively parsimonious ARCH model of TALSE returns, augmented with the concurrent neighbouring stock market information, to produce the next-day price forecasts $\hat{p}_{t+1} = \mathrm{E}(p_{t+1} \mid F_t)$, and to substitute $\hat{p}_t$ for p^f_t in (2) to compute the bubble estimate $\hat{b}_t$. There are known limitations of this methodology: firstly, the market participants are likely to have access to a much larger information set when forming their expectation of the next-day stock market price, and secondly their information set may change over time. In addition, if TALSE prices do have a bubble component, the autocorrelation that it implies will partly be washed away in the ARCH model, thereby leading to a downward bias in $\hat{b}_t$. Keeping in mind these limitations, in the absence of a more reliable statistic on the market fundamentals, the suggested method offers a reasonable compromise, if one is willing to accept that newly established stock markets like TALSE may have greater focus on the short-run speculative returns than the long-term investment gains.

The technical analysis-type market fundamentals are calculated from TALSE price forecasts based on the estimated ARCH model with 4 own lags, 3 lags of returns on Moscow, New York and London stock exchanges, and the day-of-week seasonal component. The ARCH model allows capturing changes in expectations under different volatility regimes, which is appropriate in view of the TALSE market history; refer to Section 4. However, this approach failed to produce the meaningful bubble estimates for the weekly sample, since the weekly TALSE returns appear to be unpredictable on the bases of the described information set. Therefore, weekly technical analysis-type market fundamentals coincide with the constant expectations fundamentals outlined later in this subsection.

The resulting daily $\hat{p}^f_t$ and $\hat{b}_t$ series are displayed in Figure 3. During the market upturn since late 1996 until the Russian financial crisis in mid-1998, $\hat{p}^f_t$ consistently underpredicts the actual market price, but also trends upwards, reflecting positive expectations about the future. Table 7 summarises the i.i.d. switching regression test results based on the technical analysis-type market fundamentals. The baseline model (15) has positive point estimates for $\gamma_{S,1}$ and negative for $\gamma_{C,1}$, as well as $\sigma^2_S < \sigma^2_C$ regime volatilities. Contrary to the bubble hypothesis, however, the regime switching coefficient $\gamma_{q,1}$ is positive, implying that the bubble survival probability depends positively on the bubble size. Note that most of the estimates in this model are not statistically significant at the usual 5% level.

Figure 3. Constructed ARCH-Fundamentals and ARCH-Bubble Series.



Source: Author's illustration

In fact, the LR test reported in Table 7 indicates that the null of switching volatility model (17) cannot be rejected as a suitable alternative to the full-edged bubble-linked models (15) and (16). Therefore, the technical analysis-type market fundamentals and the corresponding bubble component do not appear to have sufficient explanatory power in this test.

Table 7. I.i.d. Switching Regression with Technical Analysis-Type p_t^f

	Model (15)		Model (16)		Model (17)	
	Daily data					
$\gamma_{S,0}$	0.0010	(0.0007)	0.0008	(0.0007)	0.0008	(0.0007)
$\gamma_{S,1}$	0.0009	(0.0018)	0.0006	(0.0017)		
$\gamma_{C,0}$	-0.0050	(0.0048)				
$\gamma_{C,1}$	-0.0111	(0.0119)				
$\gamma_{q,0}$	0.7600	(0.1358)	0.8042	(0.1236)	0.8063	(0.1236)
$\gamma_{q,1}$	0.4383	(0.2211)				
$\gamma_{q,2}$	0.5000	(0.5064)				
σ^2_S	0.0149	(0.0008)	0.0147	(0.0008)	0.0147	(0.0008)
σ^2_C	0.0523	(0.0043)	0.0518	(0.0043)	0.0519	(0.0043)
L_T	1694.68		1691.52		1691.46	
LR	6.4498		0.1159		–	
T	718		718		718	

Note: Asymptotic standard errors are shown in the parentheses. The LR test is against the null of switching volatility model (17); asterisk indicates rejection at 5% level.

Source: Author's calculations

The next suggested specification of market fundamentals is based on the constant expectations hypothesis. Arguably inadequate at first sight, this approach may still provide a reasonable benchmark for approximating the future stock market expectations in a transition economy. Assuming that sizable dividend payouts to shareholders in a transition economy are likely to be backloaded far into the future because the firms need to reinvest profits, it is reasonable to view the early stock market investments being aimed mostly at speculative rather than fundamental returns. The constant expectations fundamentals, therefore, completely ignore the far-in-the-future unpredictable dividend component of stock market prices and view all short-terms market outcomes as a purely speculative process.

Table 8. I.i.d. Switching Regression with the Constant Expectations p_t^f

	Model (15)		Model (16)		Model (17)	
	Daily data					
$\gamma_{S,0}$	0.0012	(0.0010)	0.0011	(0.0010)	0.0008	(0.0007)
$\gamma_{S,1}$	-0.0002	(0.0014)	-0.0006	(0.0014)		
$\gamma_{C,0}$	0.0032	(0.0073)				
$\gamma_{C,1}$	-0.0117	(0.0101)				
$\gamma_{q,0}$	1.0130	(0.1550)	0.8061	(0.1236)	0.8063	(0.1236)
$\gamma_{q,1}$	-1.5285	(0.5165)				
$\gamma_{q,2}$	1.1332	(0.4050)				
σ^2_S	0.0144	(0.0008)	0.0147	(0.0008)	0.0147	(0.0008)
σ^2_C	0.0504	(0.0040)	0.0518	(0.0043)	0.0519	(0.0043)
L_T	1697.33		1691.57		1691.46	
LR	11.7485 *		0.2342		–	
T	718		718		718	
	Weekly data					
$\gamma_{S,0}$	0.0055	(0.0071)	0.0063	(0.0063)	0.0024	(0.0048)
$\gamma_{S,1}$	-0.0106	(0.0108)	-0.0097	(0.0093)		
$\gamma_{C,0}$	0.0150	(0.0222)				
$\gamma_{C,1}$	-0.0207	(0.0278)				
$\gamma_{q,0}$	(0.5426)	0.6547	-0.0379	(0.3280)	-0.1215	(0.3457)
$\gamma_{q,1}$	-4.5143	(2.1681)				
$\gamma_{q,2}$	2.9226	(1.4755)				
σ^2_S	0.0201	(0.0054)	0.0324	(0.0072)	0.0303	(0.0083)
σ^2_C	0.0836	(0.0058)	0.0994	(0.0122)	0.0977	(0.0117)
L_T	185.97		180.35		179.81	
LR	12.3278 *		1.0814		–	
T	145		145		145	

Note: Asymptotic standard errors are shown in the parentheses. The LR test is against the null of switching volatility model (17); asterisk indicates rejection at 5% level.
Source: Author's calculations

Table 8 shows empirical results of the i.i.d. switching regression test with $\hat{b}_t = \log p_t - \log p_1$ for the daily and weekly samples. Note that $\log p^f_t := \log p_1 \equiv \log 100$, the notional initial TALSE price, is maintained throughout the whole sample period.[16] In both samples, the point estimates of $\gamma_{S,1}$ and $\gamma_{C,1}$ are negative but statistically insignificant, and $\sigma^2_S < \sigma^2_C$, providing a weak support for the bubble hypothesis. The switching part has $\gamma_{q,2} > 0$ and $\gamma_{q,1} < 0$, resulting in U-shaped dependence on b_t. The point estimates of $\gamma_{q,2}$ and $\gamma_{q,1}$ imply that in 43% of daily observations and in 60% of weekly observations, b_t negatively affects the survival regime probability, which is consistent with the theoretical implications. However, as b_t grows larger, the estimated link flattens out, reaching plateau and turning into positive dependence for very big bubbles. The LR test rejects the null of switching volatility model (17) in favour of the fully specified alternative for both daily and weekly samples.

6. Conclusion

During the time period from June 1996 to March 1999, the Estonian stock market had witnessed some dramatic developments. The benchmark TALSE index started from the initial value of 100 in June 1996, subsequently climbing up to its three-year maximum of 492 in August 1997, just before the onset of the Asian financial crisis. After a series of big losses in the fall of 1997, and then again in the second half of 1998 in the aftermath of the Russian financial crisis, the index value went down to its initial level by the end of 1998. Is the five-fold increase of TALSE and ensuing rapid deflation of its value reminiscent of the famous "tulip mania"' and other speculative bubble episodes? The paper addresses this question using several statistical tests developed in studies of rational speculative dynamics on the Asian and American financial markets.

One important issue in applying the rational speculative bubble tests to the Estonian data is the absence of reliable stock market fundamentals, due to the absence of the dividend history. Therefore, this study relies on several direct and indirect statistical tests, where just one of the testing procedures relies on the fully specified market fundamentals. The findings are summarised below:

- The Engel and Hamilton (1990) mixture of normals model indicates the presence of two distinct stock market regimes, where one is characterised by the low-volatility positive returns and the other contains the high-volatility zero or negative returns. Persistence of the two regimes is found to be high. Although in some cases this behavior may arise from processes other than speculative bubbles, the regime switching behavior in stock market returns usually signifies that the market is exposed to some kind of speculation dynamics;
- The Hall, Psaradakis and Sola (1999) switching ADF test is another way of uncovering statistical "footprints" left by the speculative bubbles in the stock market data. The rational speculative bubbles lead to explosive roots in stock prices, and the test indeed finds that the weekly TALSE series contains a significant explosive component. However, to be sure that the unstable root in the weekly TALSE series is linked to the bubbles, one

[16] The regression equations in each regime are affine w.r.t. b_t and therefore not sensitive to this assumption. The switching part in model (15), however, is non-linear in b_t and is somewhat sensitive to alternative level assumptions. Nevertheless, $\hat{p}^f_t = \log p_1$ is maintained in this test, because any other assumption would be arbitrary.

needs to test the hypothesis using market fundamentals. Therefore, results of this test for the Estonian data remain indicative;

- In the class of direct bubble testing procedures, McQueen and Thorley (1994) duration dependence test stands out as one that does not rely on the fully specified market fundamentals. This test is applied to the runs of positive and negative TALSE returns from both daily and weekly samples. A generalisation of this testing procedure to account for a broader class of the rational speculative bubbles is also suggested in the paper. The findings indicate that, while signs of estimated coefficients do support the bubble hypothesis, out of the battery of tests most remain statistically insignificant. Circumstantial evidence points to the fact that the McQueen and Thorley (1994) duration dependence test is likely to suffer from the noisy data, but that the signs of estimated coefficients may still be a reliable clue to the presence of bubbles;
- The final test in this study is due to van Norden and Schaller (1993), and is the only test out of the four that depends on the market fundamentals. Two proxies of the fundamentals are proposed in the paper: one is based on a simple one-period forecast of the market index, and the other relies on the constant expectations hypothesis. The testing results are split: the first specification does not support the bubble hypothesis, while the second one does not reject the null of bubbles in the sample of Estonian data. A prudent conclusion from this exercise is that a rational investor should not underestimate the possibility of speculative bubbles as an explanation of the Estonian stock market history during the time period 1996-1999.

While the rational speculative bubbles provide a well-founded and empirically convenient platform for explaining the episodes akin to the one examined in this paper, other explanations of the financial asset prices rapid build-ups and sharp declines must not be disregarded. The rationality assumption behind the bubble model of Blanchard and Watson (1982) might be questioned on the grounds of a small group of investors trading on a tiny market with a limited number of assets, which is the typical picture of the Estonian stock market during the second half of the 1990s. Limited rationality models, such as the one based on the "noise trader" hypothesis of Black (1986), can provide an attractive alternative view of the Estonian stock market history amid the twin storms of the Asian and Russian financial crises. This hitherto unexplored possibility remains a potentially fruitful avenue for future research.

References

Ahmed, E., Rosser Jr, J.B. and Uppal, J.Y. 1996. Asset Speculative Bubbles in Emerging Markets: the Case of Pakistan. *Pakistan Economic and Social Review*, Vol. XXXIV, No. 2, pp. 97-188.

Black, F. 1986. Noise. *Journal of Finance*, Vol. 41, No. 3, pp. 529-43.

Blanchard, O.J. and Fischer, S. 1989. *Lectures on Macroeconomics*. Cambridge: MIT Press.

Blanchard, O.J. and Watson, M.W. 1982. Bubbles, Rational Expectations and Financial Markets. In: P. Wachel (Ed.). *Crisis in the Economic and Financial Structure*. Lexington, Mass.: Lexington Books.

Camerer, C. 1989. Bubbles and Fads in Asset Prices. *Journal of Economic Surveys*, Vol. 3, No. 1, pp. 3-41.

Cappé, O., Moulines, E. and Rydén, T. 2005. *Inference in Hidden Markov Models*. Springer.

Chan, K., McQueen G. and Thorley, S. 1998. Are There Rational Speculative Bubbles in Asian Stock Markets? *Pacific-Basin Finance Journal*, Vol. 6, No. 1-2, pp. 125-151.

Cutler, D.M., Poterba, J.M. and Summers, L.H. 1991. Speculative Dynamics. *Review of Economic Studies*, Vol. 58, No. 3, pp. 529-546.

Diba, B.T. and Grossman, H.I. 1987. On the Inception of Rational Bubbles. *Quarterly Journal of Economics,* Vol. 102, No. 3, pp. 697-700.

Dickey, D.A. and Fuller, W.A. 1981. Likelihood Ratio Statistic for Autoregressive Time Series With a Unit Root. *Econometrica*, Vol. 49, No. 4, pp. 1057-1072.

Engel, C. and Hamilton, J.D. 1990. Long Swings in the Dollar: Are They in the Data and Do Markets Know it? *American Economic Review*, Vol. 80, No. 4, pp. 689-713.

Engle, R.F. 1982. Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica*, Vol. 50, No. 4, pp. 987-1008.

Evans, G.W. 1991. Pitfalls in Testing for Explosive Bubbles in Asset Prices. *American Economic Review*, Vol. 81, No. 4, pp. 922-930.

Fama, E.F. and French, K.R. 1988. Permanent and Temporary Components of Stock Prices. *Journal of Political Economy*, Vol. 96, No. 2, pp. 246-273.

Flood, R.P. and Hodrick, R.J. 1990. On Testing for Speculative Bubbles. *Journal of Economic Perspectives*, Vol. 4, No. 2, pp. 85-101.

Flood, R.P. and Hodrick, R.J. 1986. Asset Price Volatility, Bubbles and Process Switching. *Journal of Finance*, Vol. 41, No. 4, pp. 831-842.

Goldfeld, S.M. and Quandt, R.E. 1973. A Markov Model for Switching Regressions. *Journal of Econometrics*, Vol. 1, No. 1, pp. 3-16.

Hall, S.G., Psaradakis, Z. and Sola, M. 1999. Detecting Periodically Collapsing Bubbles: a Markov-switching Unit Root Test. *Journal of Applied Econometrics,* Vol. 14, No. 2, pp. 143-154.

Hall, S.G., Urga, G. and Zalewska-Mitura, A. 1998. Testing for Evolving Stock Market Efficiency. With and Application to Russian Stock Prices. *London Business School Discussion Paper,* No. DP 12-98.

Hamilton, J.D. and Susmel, R. 1994. Autoregressive Conditional Heteroskedasticity and Changes in Regime. *Journal of Econometrics*, Vol. 64, No. 1-2. pp. 307-333.

Hamilton, J.D. 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, Vol. 57, No. 2, pp. 357-384.

Hamilton, J.D. 1990. Analysis of Time Series Subject to Changes in Regime. *Journal of Econometrics,* Vol. 45, No. 1-2, pp. 39-70.

Hartley, M.J. 1978. Estimating Mixtures of Normal Distributions. *Journal of American Statistical Association,* Vol. 73, No. 364, pp. 738-741.

Jarque, C.M. and Bera, A.K. 1987. A Test for Normality of Observations and Regression Residuals. *International Statistical Review,* Vol. 55, No. 2, pp. 163-172.

Kaplan, E.L. and Meier, P. 1958. Nonparametric Estimation from Incomplete Observations. *Journal of American Statistical Association,* Vol. 53, No. 282, pp. 457-481.

Keynes, J.M. 1936. *The General Theory of Employment, Interest and Money.* London: Macmillan.

Kim, C.-J. 1994. Dynamic Linear Models with Markov-Switching. *Journal of Econometrics,* Vol. 60, No. 1-2. pp. 1-22.

Korhonen, I. 1998. Preliminary Tests on Price Formation and Weak-form Efficiency in Baltic Stock Exchanges. *Review of Economies in Transition*, No. 5, Helsinki: Bank of Finland.

Lancaster, T. 1990. The Econometric Analysis of Transition Data. *Econometric Society Monographs,* No. 17, Cambridge University Press.

Ljung, G.M. and Box, G.P.E. 1978. On a Measure of Lack of Fit in Time Series Models. *Biometrika*, Vol. 65, No. 2, pp. 297-303.

McQueen, G. and Thorley, S. 1994. Bubbles, Stock Returns, and Duration Dependence. *Journal of Financial and Quantitative Analysis,* Vol. 29, No. 3, pp. 379-401.

McQueen, G. and Thorley, S. 1991. Are Stock Returns Predictable? A Test Using Markov Chains. *Journal of Finance*, Vol. 46, No. 1, pp. 239-263.

Poterba, J.M. and Summers L.H. 1988. Mean Reversion in Stock Prices: Evidence and Implications. *Journal of Financial Economics,* Vol. 22, pp. 27-59.

Reinhart, C.M. and Rogoff, K.S. 2009. *This Time is Different. Eight Centuries of Financial Folly.* Princeton University Press.

Schwert, W.G. 1990. Stock Volatility and the Crash of '87. *Review of Financial Studies*, Vol. 3, No. 1, pp. 77-102.

Shields, K. 1997. *Stock Return Volatility on Emerging Eastern European Markets.* The Manchester School Supplement, pp. 118-138.

Shiller, R. 1981. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review,* Vol. 71, No. 3, pp. 421-436.

Taylor, S.J. 2005. *Asset Price Dynamics, Volatility, and Prediction.* Princeton University Press.

Tirole, Jean. 1982. On the Possibility of Speculation Under Rational Expectations. *Econometrica*, Vol. 50, No. 5, pp. 1163-1181.

Tirole, J. 1985. Asset Bubbles and Overlapping Generations. *Econometrica*, Vol. 53, No. 6, pp. 1071-1100.

van Norden, S. 1996. Regime Switching as a Test for Exchange Rate Bubbles. *Journal of Applied Econometrics,* Vol 11, No. 3, pp. 219-251.

van Norden, S. and Schaller, H. 1993. The Predictability of Stock Market Regime: Evidence from the Toronto Stock Exchange. *The Review of Economics and Statistics,* Vol. LXXV, No. 3, pp. 505-510.

van Norden, S. and Schaller, H. 2002. Fads or Bubbles? *Empirical Economics,* Vol. 27, pp. 335-362.

Weil, P. 1990. On the Possibility of Price Decreasing Bubbles. *Econometrica*, Vol. 58, No. 6, pp. 1467-1474.

West, K.D. 1987. A Specification Test for Speculative Bubbles. *Quarterly Journal of Economics,* Vol. 102, No. 3, pp. 553-580.

Zalewska-Mitura, A. and Hall, S.G. .1998. Examining the First Stages of Market Performance. A Test for Evolving Market Efficiency. *London Business School Discussion Paper*, No. DP 11-98.